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No. A-262

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Probability Lab

Instructional Computing Courseware
for **Apple®** II Series Computers

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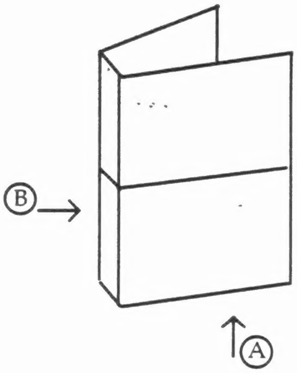
How to use your MECC folder cover inserts



Tear cover on perforated lines. Place half-page title section in the clear vinyl pocket Ⓐ on the front of the binder. Insert finger-sized title in the clear plastic pocket on the spine Ⓑ.

Probability Lab

No. A-262
For Apple® II Series Computers
Grades 7-12

Design probability experiments using coins, dice, cards, spinners, and numbers.

Probability Lab

This manual is compatible

with

the ***Probability Lab*** disk

Version 1.x

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3490 Lexington Avenue North
St. Paul, MN 55126

ISBN 0-7929-0149-5 June 1990

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Library of Congress Cataloging-in-Publication Data

Probability lab [computer file] . — Version 1.0.

1 computer disk : sd., col. ; 3 1/2 in. + 1 manual.
System requirements: Apple II series; 128K RAM; ProDOS; BASIC; 1 disk drive; monochrome or color monitor; printer (recommended).
Title from title screen.
Edition statement from disk label.
Copy-protected.
Audience: Grades 7-12.
Issued also on 5 1/4 in. computer disk.
Summary: A utility program that enables students to model problems found in textbooks, to investigate concepts or rules of probability, and to test ideas involving probability. Includes six experiments for use in probability lessons.
Contents: Flip coins—Roll dice—Draw cards—[etc.]
"A-262"—Disk label.
1. Probabilities—Software. [1. Probabilities—Software]
I. Minnesota Educational Computing Corporation.
QA273.19.E4P76 1990 519.2 90-69819
ISBN 0-7929-0149-5

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THE PROGRAM AT A GLANCE

The study of probability has become a very important part of mathematics. While the origins of sixteenth-century formal probability were rooted in gambling, the concepts and rules of probability have become essential tools not only for today's decision makers in business, industry, and government, but also for each one of us during the daily conduct of our life. Whether preparing an annual budget for a large corporation based on past performance and market research or planning a weekend outing based on weather reports, probability plays a significant role in wise decision-making.

Probability Lab is a learning tool that can help students develop concepts and test different hypotheses related to the study of probability. Providing students with opportunities to experiment with probability and design models simulating probability problems helps students distinguish between experimental probability and theoretical probability and learn basic probability concepts by using realistic models.

Doing real experiments with real objects is both important and fun for students; it is often difficult, however, to conduct enough trials to gain an accurate sense of the probability that a given event will occur. For example, flipping a single coin 100 times and recording how many times "heads" occurred would be a relatively simple experiment for most students to complete. But flipping ten coins 100 times and recording how many times exactly "seven heads and three tails" occurred would be both difficult and time-consuming. This is where ***Probability Lab*** can be quite helpful. Not only will students be able to design more complex experiments and gather data from many trials in a short amount of time, but they will also be able to look for trends and relationships in a series of related probability experiments. In fact, by using an Apple IIGS set at fast speed, it is possible to accurately simulate 1,000,000 flips of a single coin and record the results in less than fifteen minutes!

Probability Lab provides both variety and flexibility. Many of the experiments suggested for use in probability lessons usually involve events with either two or six possible outcomes in order to allow you to use coins or dice to simulate the problem. With ***Probability Lab***, you are able to use up to ten coins or dice, a standard deck of playing cards, up to five wheels (or spinners as they are sometimes referred to), a bag of up to 100 assorted marbles, or a number board containing up to 100 numbers. For added flexibility, you can even customize some of these items. For example, your dice can have from two to twenty faces and the wheels (spinners) can have up to twenty items per wheel, selected from six different objects.

Probability Lab is a utility program and can be used by individual students, small groups, or a large group with the aid of a projection system. It enables students to model problems found in textbooks, investigate concepts or rules of probability, and test ideas involving probability. Among the many features found in this package are:

- the choice to use coins, dice, cards, wheels, marbles, and numbers when designing an experiment;
- the ability to customize dice, spinners, marbles, and numbers;
- automatic success-checking for most experiments;
- the ability to set the number of trials per experiment to 1,000,000;
- an opportunity to predict success;
- a graphical "step mode" to witness individual trials;
- an "auto mode" for speed;
- an opportunity to see the expected success;
- the ability to record the first 100 trials of your experiment for further analysis; and,
- teacher options to regulate experiment availability and the use of predicted and expected success.

OBJECTIVES

After using ***Probability Lab***, users will have had an opportunity to:

- solve probability problems experimentally;
- construct probability models to replicate real situations;
- predict the probability of success for an experiment;
- compare predictions to actual successes for experiments; and,
- conduct various probability investigations.

EQUIPMENT REQUIRED

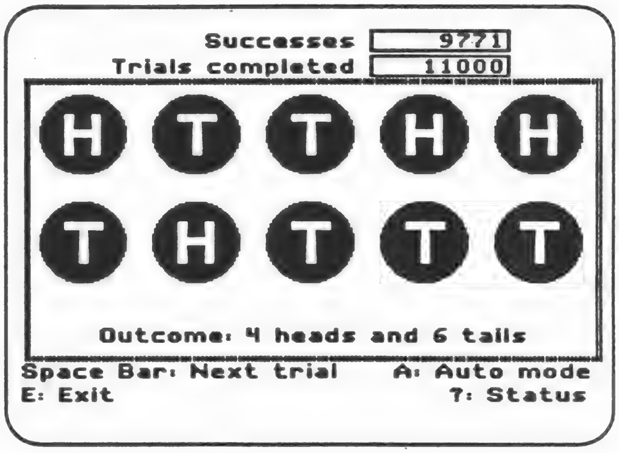
The hardware requirements for using ***Probability Lab*** are:

- an Apple II series computer with at least 128K of memory;
- a single disk drive (3.5" or 5.25");
- a monochrome or color monitor; and
- a printer (recommended, but not required).

A QUICK OVERVIEW OF EXPERIMENT TYPES

Coin Experiments

- Flip from 1 to 10 coins per trial
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



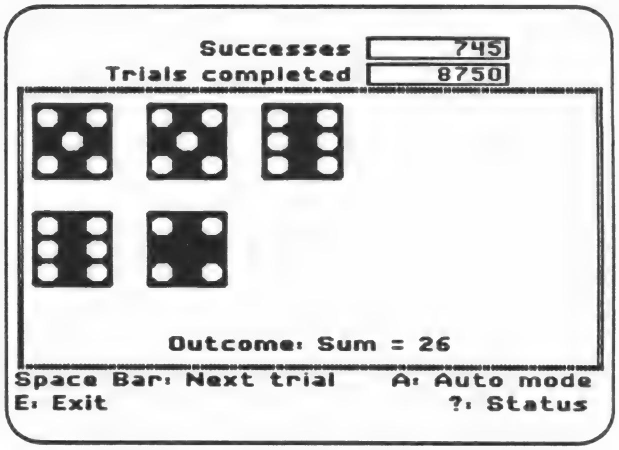
Successes 9771
Trials completed 11000

Outcome: 4 heads and 6 tails

Space Bar: Next trial A: Auto mode
E: Exit ?: Status

Dice Experiments

- Roll from 1 to 10 dice per trial
- Customize dice to have from 2-20 faces
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



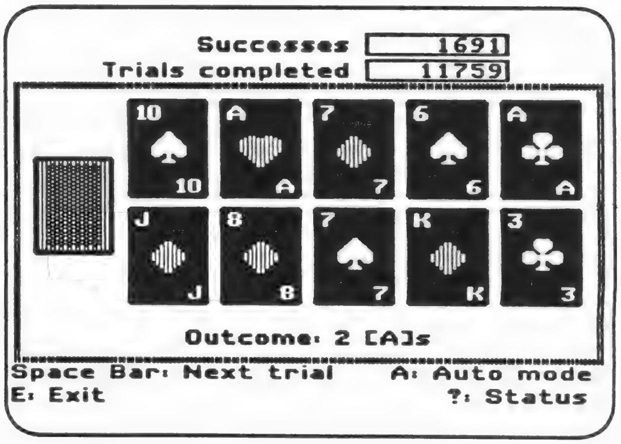
Successes 745
Trials completed 8750

Outcome: Sum = 26

Space Bar: Next trial A: Auto mode
E: Exit ?: Status

Card Experiments

- Draw from 1 to 10 cards per trial
- Reshuffle or discard after each draw
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



Successes 1691
Trials completed 11759

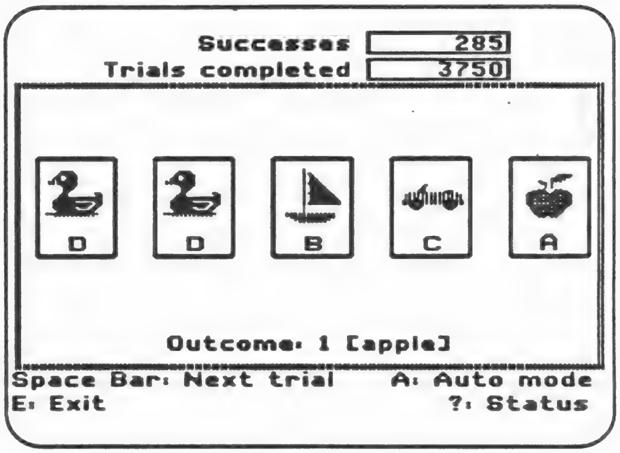
Outcome: 2 [A]s

Space Bar: Next trial A: Auto mode
E: Exit ?: Status

A QUICK OVERVIEW OF EXPERIMENT TYPES (continued)

Wheel Experiments

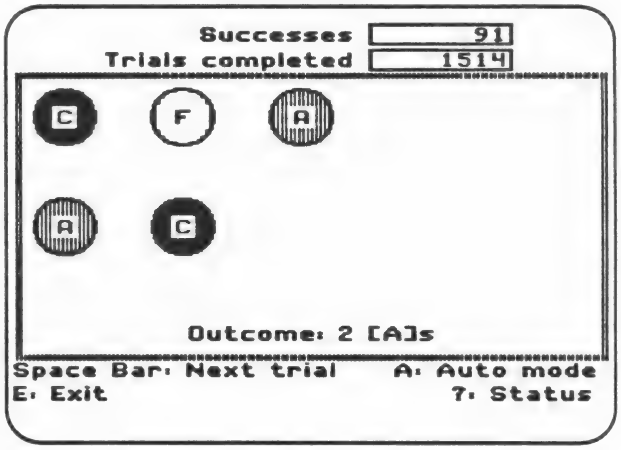
- Spin from 1 to 5 wheels per trial
- Place up to 20 objects on a wheel
- 6 different types of objects available
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



```
            Successes   285
      Trials completed  3750

   D      D      B      C      A

        Outcome: 1 [apple]
Space Bar: Next trial    A: Auto mode
E: Exit                    ?: Status
```

Marble Experiments

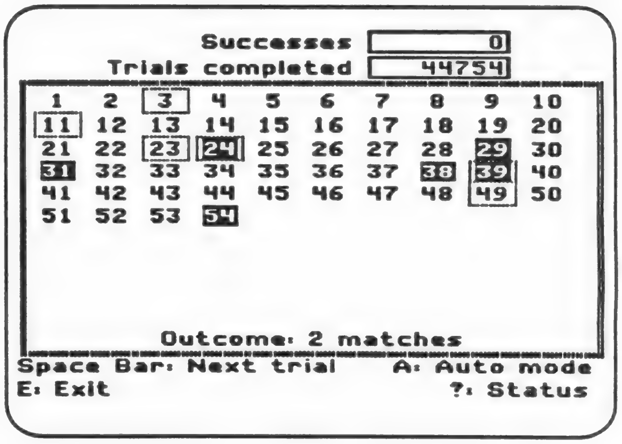
- Draw from 1 to 10 marbles per trial
- Marble bag can hold up to 100 marbles
- 6 different types (colors) of marbles
- Put back or leave out marbles after each draw
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



```
            Successes    91
      Trials completed  1514

   C      F      A

   A      C

          Outcome: 2 [A]s
Space Bar: Next trial    A: Auto mode
E: Exit                    ?: Status
```

Number Experiments

- Pick from 1 to 10 numbers per experiment
- Computer picks up to 10 numbers per trial
- Use up to 100 numbers in your experiment
- Up to 1,000,000 trials in an experiment
- Success can be defined in many ways
- Visual "step mode" or fast "auto mode"
- Option to predict success available
- Option to see expected success available



```
            Successes     0
      Trials completed  44754
  1   2   3   4   5   6   7   8   9  10
 11  12  13  14  15  16  17  18  19  20
 21  22  23  24  25  26  27  28  29  30
 31  32  33  34  35  36  37  38  39  40
 41  42  43  44  45  46  47  48  49  50
 51  52  53  54

        Outcome: 2 matches
Space Bar: Next trial    A: Auto mode
E: Exit                    ?: Status
```

THINKING SKILLS

Using Computer Software in a Thinking Skills Environment

Teachers are faced with the tremendous task of preparing today's students for tomorrow's world—a world characterized by change in an information-rich environment. Thinking skills are at the heart of this thriving, changing environment, for these are the behaviors students must practice in school and continue to apply for the rest of their lives.

It wasn't long ago that thinking skills were considered exclusive to gifted and enrichment classes. Today, however, thinking skills are viewed as an essential component of the total school curriculum. Developing these skills is the goal of each individual discipline. Many educators have, in fact, come to view thinking skills as perhaps the most basic of the basic skills because they facilitate the acquisition of all other learning.

At MECC, we view computer software as a vehicle for fostering students' thinking. Our products are curriculum-based, with thinking skills as a thread within subject areas. This provides an environment with many opportunities for teachers to highlight and reinforce thinking skills.

We believe teachers play a critical role in determining the classroom environment for thinking. Naturally, many teachers have taught thinking skills and will continue to do so using a variety of strategies. Our commitment is to provide teachers with the materials that help them do their job well: high-quality software that promotes the application of thinking skills.

Our approach to thinking skills reflects what both research and effective classroom practice has shown. That is, the approach that is most effective and appeals to most teachers is one that infuses thinking skills into existing content areas. Educators have told us they are interested in thinking skills as a method used in the instruction of a topic, not as a subject. By infusing thinking skills into existing content areas, MECC products integrate easily into teachers' curricula while providing a rich environment for students to practice skillful thinking. We strive to meet the challenge teachers face in promoting the skills that students need.

If schools are to integrate the teaching of thinking with regular academic instruction, they need to know what aspects of thinking to teach. After exploring the research that has been done in the area of thinking skills, MECC has chosen as a base the *Dimensions of Thinking* framework, published in 1988 by the Association of Supervision and Curriculum Development (ASCD). We chose this framework because it pulls together research and models from a variety of sources and brings the theory to the classroom level, applying it to that environment. In addition to knowing the subject matter that is covered, teachers now can see the specific thinking skills that are challenged within a product.

This section of the manual highlights ways in which teachers can use ***Probability Lab*** to promote thinking skills with their students. The following pages provide examples of how ***Probability Lab*** relates to the ASCD core thinking skills framework. Although only one thinking skill per category is correlated to a specific part of the product, each skill can be practiced on many levels and in many aspects of the product.

We realize the importance of thinking skills in the curriculum. We believe it is essential that students be taught thinking skills so that they have the tools to understand the past, deal with the present, and prepare for the future. We are confident that you will find ***Probability Lab*** of considerable value in your classroom as you foster student thinking.

A Framework for Thinking

The components used in thinking are referred to as *core thinking skills*. This framework defines those skills that appear in the repertoire of the model learner. Each skill selected is documented in research as important to learning or thinking, is teachable, and is valued by educators as important for students to learn.

The core skills of the ASCD framework are listed and defined below with examples of applications within ***Probability Lab.*** The skills are neither discrete nor hierarchical. In fact, individual skills draw on other skills and can be used repeatedly in the thinking process. The selected examples are not exhaustive but highlight ways in which these thinking skills are used in ***Probability Lab.***

Source: Dimensions of Thinking, Association for Supervision and Curriculum Development (ASCD), 1988.

Definition of Core Thinking Skills Categories	Core Thinking Skills Components	*Probability Lab* Application
Focusing Skills allow students to attend to selected pieces of information and ignore others. Focusing occurs when students sense a problem, an issue, or a lack of meaning.	**Focusing Skills** • Defining Problems • Setting Goals	In ***Probability Lab***, students select from among the six different probability experiments available to conduct. They determine the components of their investigation and define the criteria they will use to determine success.
Information Gathering Skills involve obtaining information and clarifying issues and meanings through inquiry.	**Information Gathering Skills** • Observing • Formulating Questions	As each trial is conducted in a probability experiment, students must skillfully observe the number of times each trial meets their definition of success. Students begin to inquire about how factors such as the number of trials and the criteria for success influences their prediction.
Remembering Skills are those activities or strategies that students consciously engage in to store and retrieve information from long-term memory. Activating prior knowledge falls under this category.	**Remembering Skills** • Encoding • Recalling	Each experiment students conduct allows them to add to their existing knowledge base of information on probability. Students will draw on this knowledge of probability as they restructure their investigations in order to make accurate predictions.

THINKING SKILLS (continued)

Definition of Core Thinking Skills Categories	Core Thinking Skills Components	*Probability Lab* Application
Organizing Skills are used to arrange information so that it can be understood or presented more effectively.	**Organizing Skills** • Comparing • Classifying • Ordering • Representing	After completing a prediction for a given experiment, students have the opportunity to compare their prediction to the actual rate of success. Students can also compare the theoretical probability of an event occurring with the experimental probability.
Analyzing Skills are used to clarify existing information by examining parts and relationships. Through analysis, students identify and distinguish components, attributes, claims, assumptions, or reasoning.	**Analyzing Skills** • Identifying Attributes and Components • Identifying Relationships and Patterns • Identifying Main Ideas • Identifying Errors	Students can look for trends and relationships in a series of related probability experiments. ***Probability Lab*** retains the first 100 trials of an experiment for further analysis.
Generating Skills involve using the students' prior knowledge to add information beyond what is given. Connections between new ideas and prior knowledge are made as new information and ideas are recast into new structures.	**Generating Skills** • Inferring • Predicting • Elaborating	Before actually starting an experiment in ***Probability Lab***, the students will make a prediction regarding the experiment's rate of success. Students can predict success by entering a percent or the actual number of successes.
Integrating Skills involve putting together the relevant parts or aspects of a solution, understanding, principle, or composition and incorporating this integrated information into a new understanding.	**Integrating Skills** • Summarizing • Restructuring	Students can structure other investigations and simulate the probability of an event by designing an appropriate model using ***Probability Lab***. Investigations could simulate real-life experiences as students integrate the information they have already acquired.
Evaluating Skills involve assessing the reasonableness and quality of ideas.	**Evaluating Skills** • Establishing Criteria • Verifying	At this point, students can share the process and strategies they employed to arrive at their prediction.

A SAMPLE EXPERIMENT USING *PROBABILITY LAB*

Before using ***Probability Lab*** with students, you should first familiarize yourself with the operation of the package and the features available. In some cases, you may discover that certain features can be regulated through the use of Teacher Options (see pages 16-19) in order to better meet your needs and those of your students. This section takes you through the steps of a simple probability experiment to illustrate the process. It may be helpful to actually use the ***Probability Lab*** program while following the steps.

Let's take the traditional probability experiment of flipping some coins and, to show some of the power of ***Probability Lab***, of flipping more than just a single coin. The procedure used in the following sample experiment will be basically the same for other types of probability experiments:

> **Flip 8 coins at a time and find the probability that exactly 4 of the coins will be heads. Repeat this 10,000 times and record the results.**

Under normal circumstances, the above experiment would require many hours of tedious coin flipping. Now let's see how it can be accurately simulated in just a few minutes!

Step 1: Place the ***Probability Lab*** disk in the disk drive and turn on the computer. If accessing the package on a network, follow the instructions for your particular setup. After a few seconds, the main menu screen (Figure 1) will appear. Select the first menu item "Flip Coins" by pressing the Return Key.

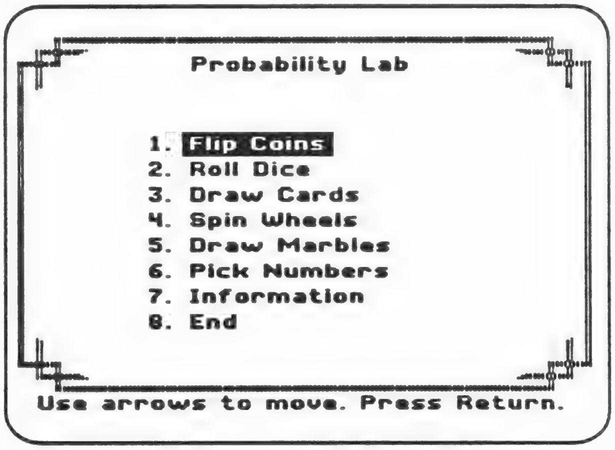
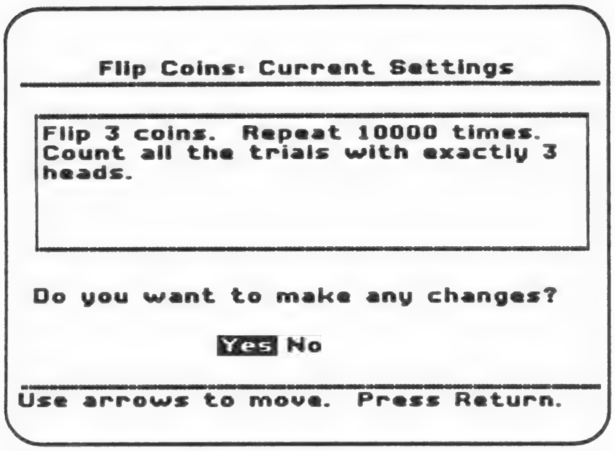


Figure 1

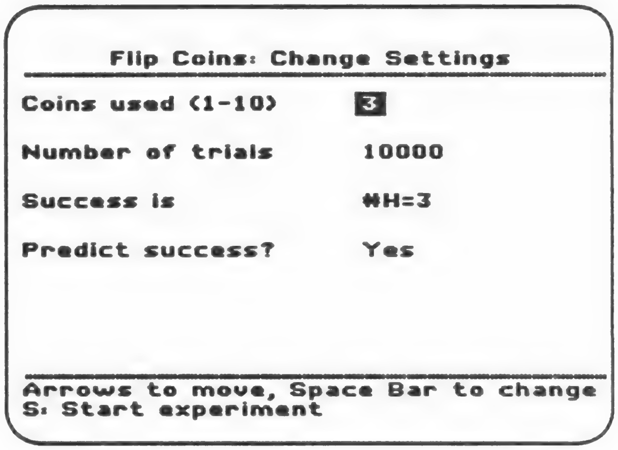
A SAMPLE EXPERIMENT USING *PROBABILITY LAB* (continued)

Step 2: Each type of experiment has a pre-set design with its current setting shown on the opening screen (Figure 2). After reading the description of this experiment, you will realize that changes must be made in order to run our sample experiment. Select "Yes" and a menu of choices related to flipping coins will appear (Figure 3).



```
Flip Coins: Current Settings

Flip 3 coins.  Repeat 10000 times.
Count all the trials with exactly 3
heads.

Do you want to make any changes?

          Yes No

Use arrows to move.  Press Return.
```

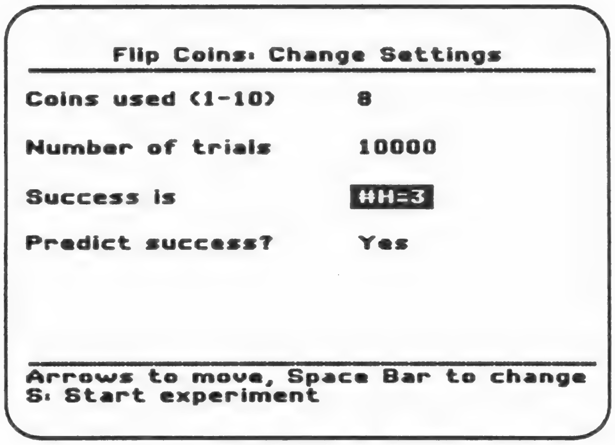
Figure 2



```
Flip Coins: Change Settings

Coins used (1-10)       3

Number of trials        10000

Success is              #H=3

Predict success?        Yes

Arrows to move, Space Bar to change
S: Start experiment
```

Figure 3

Step 3: Make the appropriate changes to the current settings (Figure 4). If you are uncertain as to which keys to use in moving around and making changes, note the instructions at the bottom of the screen. Change the number of coins used to 8. Then redefine success for the experiment so that the program checks for all trials in which exactly 4 heads occur. Before doing this, however, a brief explanation might be helpful.



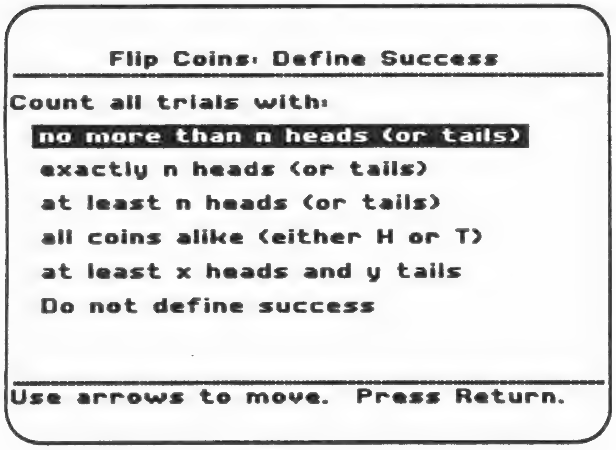
```
Flip Coins: Change Settings

Coins used (1-10)       8

Number of trials        10000

Success is              #H=3

Predict success?        Yes

Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4

A SAMPLE EXPERIMENT USING *PROBABILITY LAB* (continued)

Defining success and checking for it during an experiment is one of the most important aspects of the experiment. After all, of what value would it be to merely throw our 8 coins into the air and pick them up 10,000 times? The goal in this experiment is to determine in how many of the 10,000 trials exactly 4 heads will occur. In order to accomplish this, one must carefully inspect each trial and determine whether or not it was a success with exactly 4 heads. But this is a tedious task and one that can easily be handled by the computer. Therefore, define success and let the computer do the checking and counting.

Step 4: Place the cursor (inversed area) on the current definition of success (#H = 3) and press Return. You will then see a list of the different ways in which success can be defined for any coin experiment (Figure 5). Since you are interested in counting all trials in which *exactly 4 heads* appear, choose the second option in the list. Choosing this option will result in two follow-up questions (Figure 6). The first will ask if you are looking for "heads" or "tails." The second will ask how many (n) heads you are looking for.


Flip Coins: Define Success

Count all trials with:

no more than n heads (or tails)

exactly n heads (or tails)

at least n heads (or tails)

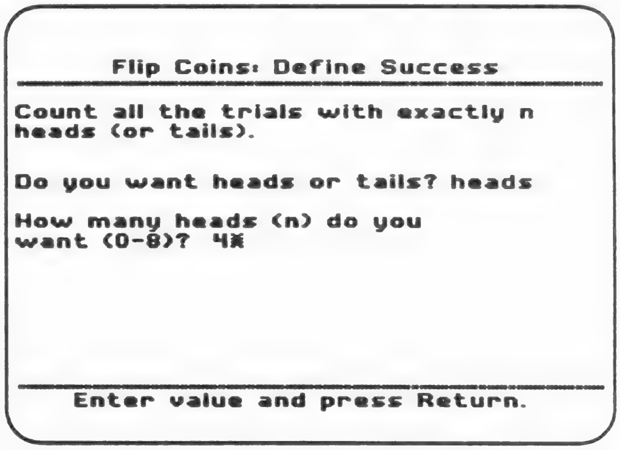
all coins alike (either H or T)

at least x heads and y tails

Do not define success

Use arrows to move. Press Return.

Figure 5


Flip Coins: Define Success

Count all the trials with exactly n heads (or tails).

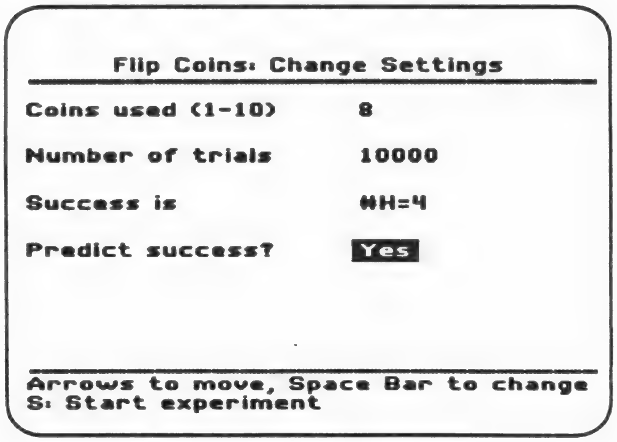
Do you want heads or tails? heads

How many heads (n) do you want (0-8)? 4

Enter value and press Return.

Figure 6

Step 5: Leave the final setting (Figure 7) regarding "Predict success" set on "Yes." This simply means that before starting the actual experiment you will make a prediction as to what you think the rate of success will be for the experiment. Of course, you will want to do this in order to see how close you can get! Just press the S Key to start the experiment.


Flip Coins: Change Settings

Coins used (1-10) 8

Number of trials 10000

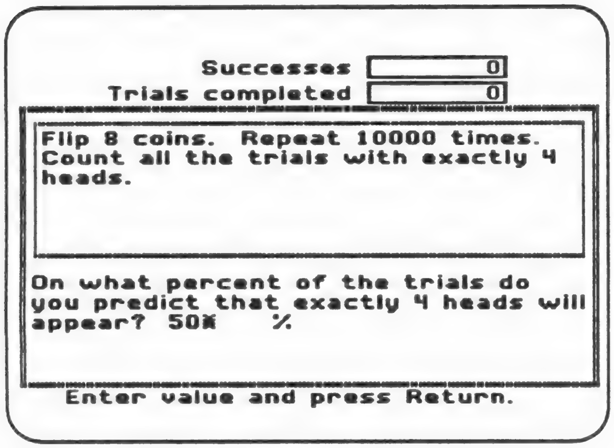
Success is #H=4

Predict success? Yes

Arrows to move, Space Bar to change
S: Start experiment

Figure 7

Step 6: Our sample experiment was given to a group of junior high students during the testing phase of this package. Most of the students were certain that, because heads occur about 50% of the time in all coin flips, they would get exactly 4 heads about 50% of the time in this experiment. Consequently, 50% will be used as the prediction for this example (Figure 8). Of course, you can feel free to enter what ever you like. Once you enter your prediction, it will be translated to raw numbers (Figure 9) and you can determine if it sounds reasonable. If not, you will be given another opportunity to predict.

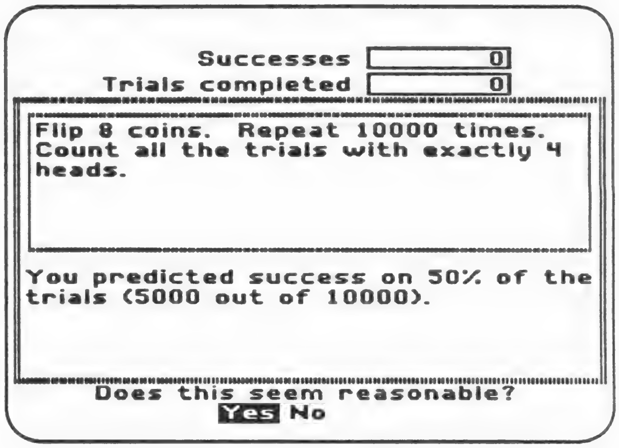

Successes 0
Trials completed 0

Flip 8 coins. Repeat 10000 times.
Count all the trials with exactly 4 heads.

On what percent of the trials do you predict that exactly 4 heads will appear? 50▮ %

Enter value and press Return.

Figure 8


Successes 0
Trials completed 0

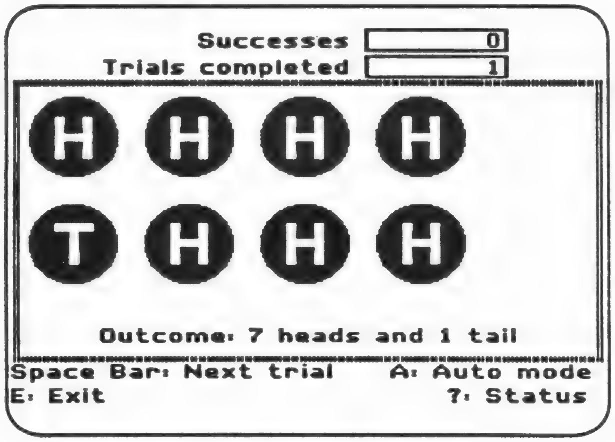
Flip 8 coins. Repeat 10000 times.
Count all the trials with exactly 4 heads.

You predicted success on 50% of the trials (5000 out of 10000).

Does this seem reasonable?

Figure 9

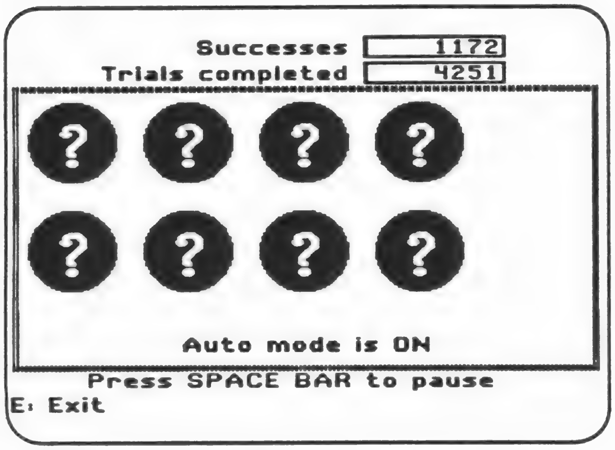
Step 7: It is now time to run the actual experiment. Start by pressing the Space Bar to see the next trial (in this case it is actually the first trial). Pressing the Space Bar will put you in a graphical "step mode" so that you can see each trial (Figure 10). Step through a few trials and note the two counters at the top of the screen. The top counter keeps track of all successful trials, while the second one keeps track of the total trials completed. Pressing the ? Key will also give you the current settings and current rate of success for the experiment.


Successes 0
Trials completed 1

Outcome: 7 heads and 1 tail

Figure 10

Step 8: Once you have assured yourself that the right number of coins are being used and that the computer is doing a good job of keeping track of successes, shift into "auto mode" by pressing the A Key and watch as the experiment is quickly simulated (Figure 11). The counters will be updated every 250 trials so that you can keep an eye on the progress. If you wish to revert back to step mode, just press the Space Bar. Within a few seconds, the experiment will be completed (Figure 12).

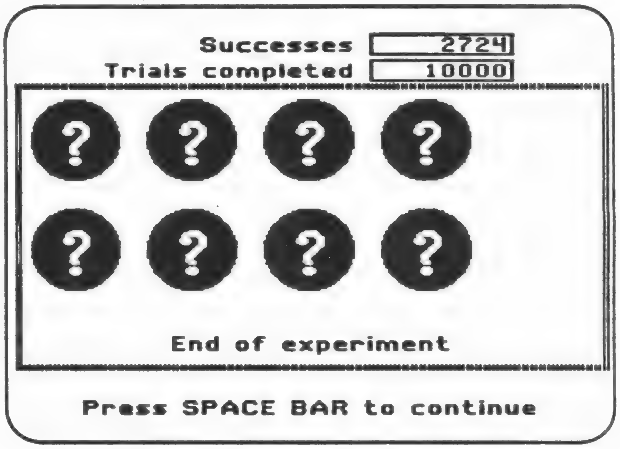


Successes 1172
Trials completed 4251

Auto mode is ON

Press SPACE BAR to pause
E: Exit

Figure 11



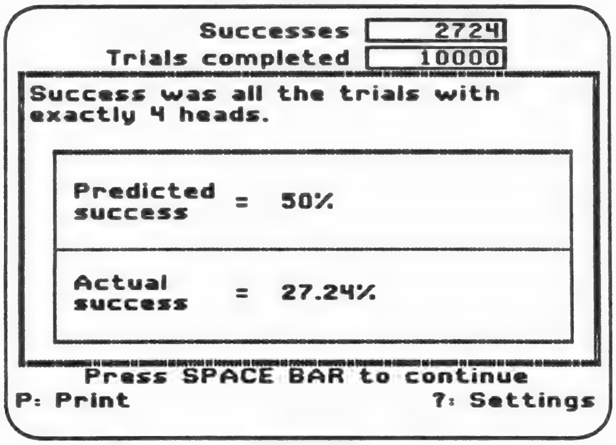
Successes 2724
Trials completed 10000

End of experiment

Press SPACE BAR to continue

Figure 12

Step 9: A summary screen will compare your prediction with the actual success of the experiment (Figure 13). If you have a printer attached, this summary can be printed by pressing the P Key. A review of the settings for the experiment can also be accessed by pressing the ? Key.



Successes 2724
Trials completed 10000

Success was all the trials with exactly 4 heads.

Predicted success = 50%

Actual success = 27.24%

Press SPACE BAR to continue
P: Print ?: Settings

Figure 13

Step 10: The final menu screen allows you to go back and run another coin experiment or to view some of the results (Figure 14). While it would not be possible to save the actual outcomes for all 10,000 trials, the first 100 trials from each experiment are stored. This "sample space" (as it is often referred to in probability books) can be viewed or printed as an example for use in further study (Figure 15).

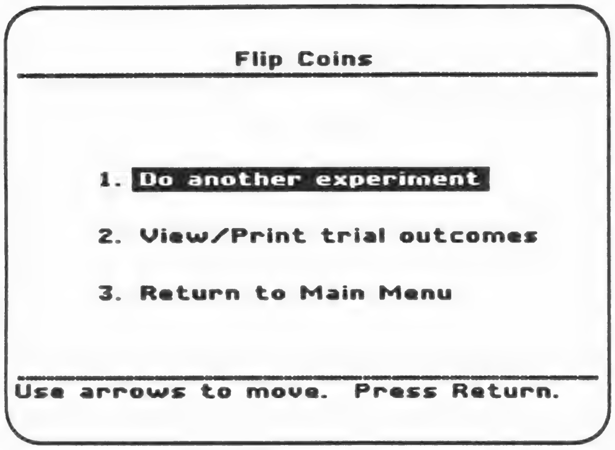
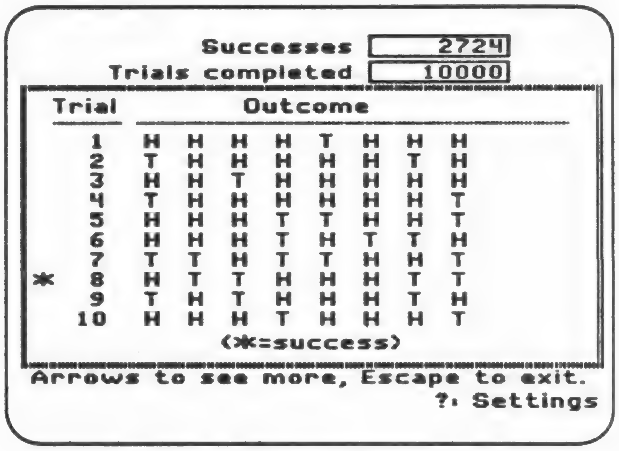

Flip Coins
1. Do another experiment
2. View/Print trial outcomes
3. Return to Main Menu
Use arrows to move. Press Return.

Figure 14



```
                 Successes [  2724]
         Trials completed [ 10000]
  Trial           Outcome
     1   H  H  H  H  T  H  H  H
     2   T  H  H  H  H  H  T  H
     3   H  H  T  H  H  H  H  H
     4   T  H  H  H  H  H  H  T
     5   H  H  H  T  T  H  H  T
     6   H  H  H  T  H  T  T  H
     7   T  T  H  T  T  H  H  T
*    8   H  T  T  H  H  H  T  T
     9   T  H  T  H  H  H  T  H
    10   H  H  H  T  H  H  H  T
             (*=success)
Arrows to see more, Escape to exit.
                        ?: Settings
```

Figure 15

What's next? Perhaps you are wondering what would happen if you ran the experiment 100,000 times. Maybe you want to try a similar experiment using a different number of coins. Or perhaps you are curious about a different type of experiment. Spend some time exploring and try the many different options available. If you are uncertain how some features work or want to know more about some of them, consult the appropriate section of the manual.

Using *Probability Lab* in the Classroom

Teacher Options 16
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    Regulate Experiment Design 17
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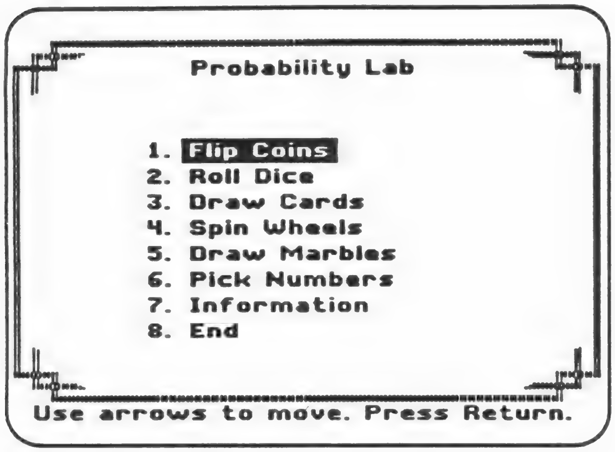
TEACHER OPTIONS

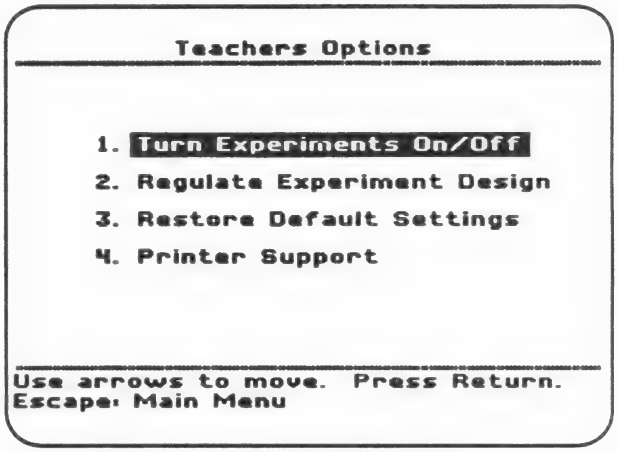
Probability Lab has some special options that allow teachers to regulate some of the features contained in the package. To access the Teacher Options, press Control-A while viewing the main menu screen (Figure 1). The menu of Teacher Options will appear (Figure 2).


Probability Lab

1. Flip Coins
2. Roll Dice
3. Draw Cards
4. Spin Wheels
5. Draw Marbles
6. Pick Numbers
7. Information
8. End

Use arrows to move. Press Return.

Figure 1


Teachers Options

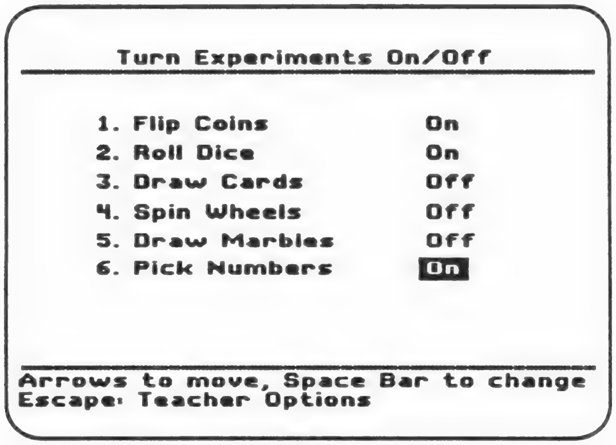
1. Turn Experiments On/Off
2. Regulate Experiment Design
3. Restore Default Settings
4. Printer Support

Use arrows to move. Press Return.
Escape: Main Menu

Figure 2

Turn Experiments On/Off

The first option allows you to turn any of the experiments on or off (Figure 3). This enables you to select only those investigations that match the goals of your curriculum and the age and ability of your students. It can also be used in a situation where you want only a few experiments available at a time in order to properly sequence a unit of study or to ensure that students are using the desired experiments. At least one of the experiments must be left on.


Turn Experiments On/Off

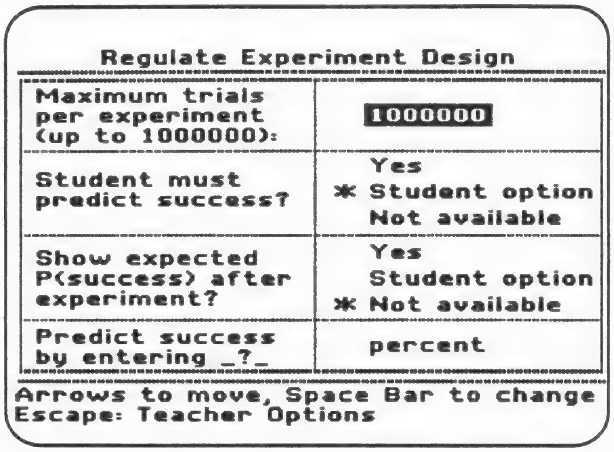
1. Flip Coins On
2. Roll Dice On
3. Draw Cards Off
4. Spin Wheels Off
5. Draw Marbles Off
6. Pick Numbers On

Arrows to move, Space Bar to change
Escape: Teacher Options

Figure 3

Regulate Experiment Design

The second option allows you to regulate some of the variables used in designing an experiment (Figure 4).



Regulate Experiment Design

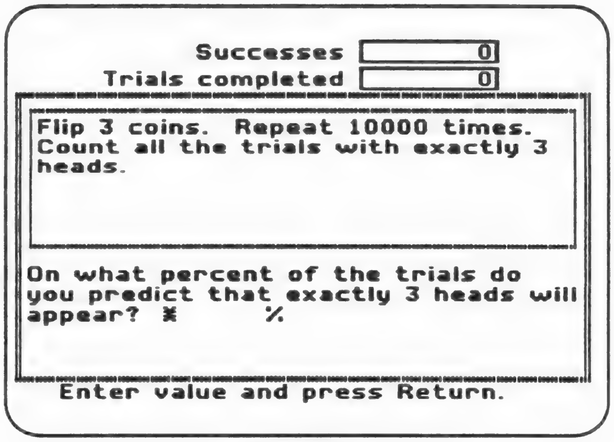
Maximum trials per experiment (up to 1000000):	1000000
Student must predict success?	Yes ✱ Student option Not available
Show expected P(success) after experiment?	Yes Student option ✱ Not available
Predict success by entering _?_	percent

Arrows to move, Space Bar to change
Escape: Teacher Options

Figure 4

The maximum number of trials per experiment can be set anywhere in the range of from 1 to 1,000,000 trials. If the amount of time spent on an individual experiment is an important consideration, set this maximum to the appropriate limit. Charts are provided in each section to help you in determining the approximate time required in various situations for each type of experiment.

Prior to the start of any experiment, the student can be asked to predict the rate of success for the experiment (Figure 5). If you want this to automatically happen, just check the "Yes" option. If you want this to be an option for the student, select "Student option." If you prefer that predicting success is not available, just select the "Not available" option.



Successes 0
Trials completed 0

Flip 3 coins. Repeat 10000 times. Count all the trials with exactly 3 heads.

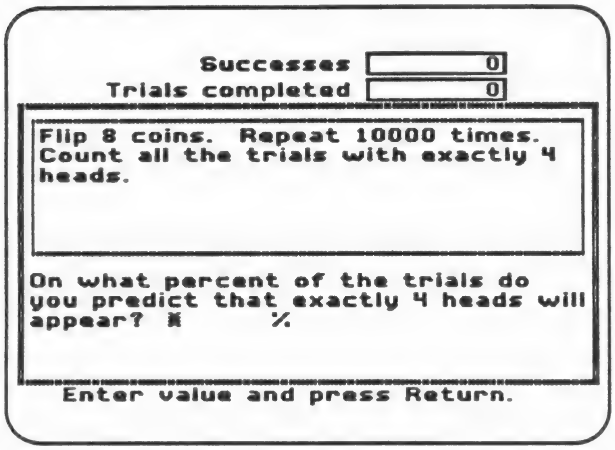
On what percent of the trials do you predict that exactly 3 heads will appear? ✱ %

Enter value and press Return.

Figure 5

TEACHER OPTIONS (continued)

There are two ways in which students can enter their success predictions. Probably the easiest method is to enter a percent (Figure 6). There may, however, be situations where it would be more beneficial to enter the actual number of successes (Figure 7). You can set this option to either method.



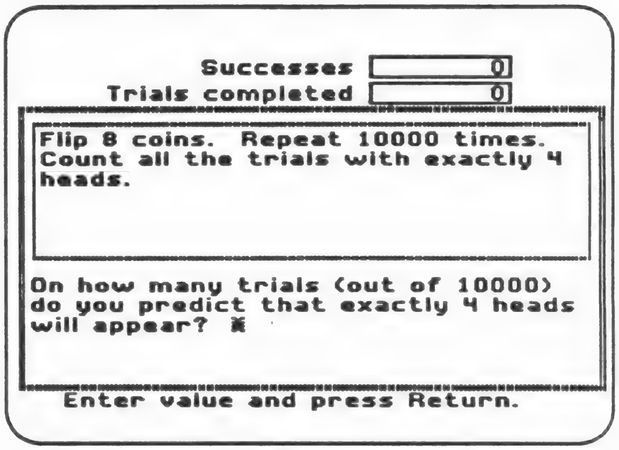
Successes 0
Trials completed 0

Flip 8 coins. Repeat 10000 times. Count all the trials with exactly 4 heads.

On what percent of the trials do you predict that exactly 4 heads will appear? %

Enter value and press Return.

Figure 6



Successes 0
Trials completed 0

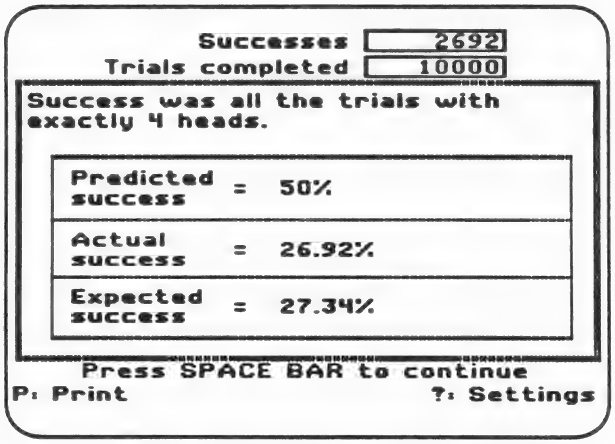
Flip 8 coins. Repeat 10000 times. Count all the trials with exactly 4 heads.

On how many trials (out of 10000) do you predict that exactly 4 heads will appear?

Enter value and press Return.

Figure 7

You can also regulate whether or not students are shown the expected probability of success [denoted as P (success)] after each experiment. Figure 8 illustrates this feature. By default, this option is not available. You will need to determine if this feature is in line with the goals of your lessons. If you are trying to get your students to develop an intuitive feeling for probability, you would not want to use this feature. If you were, however, trying to illustrate the differences between actual and expected outcomes, you would want to use this feature.



Successes 2692
Trials completed 10000

Success was all the trials with exactly 4 heads.

Predicted success	=	50%
Actual success	=	26.92%
Expected success	=	27.34%

Press SPACE BAR to continue

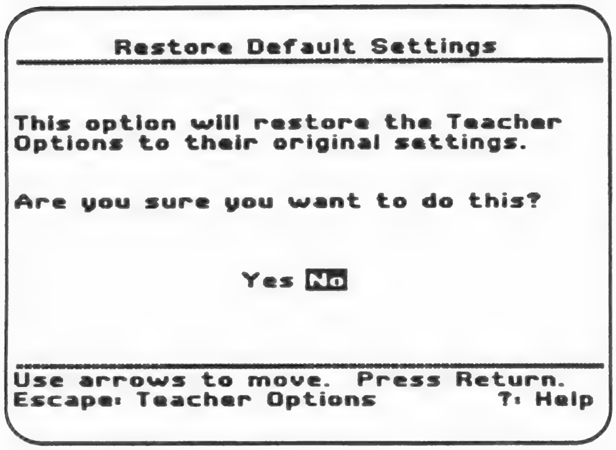
P: Print ?: Settings

Figure 8

TEACHER OPTIONS (continued)

Restore Default Settings

You can easily restore the default settings for the Teacher Options found on your disk (Figure 9). A summary of the default settings can be seen by pressing the ? Key. Figure 10 shows these settings.



Restore Default Settings

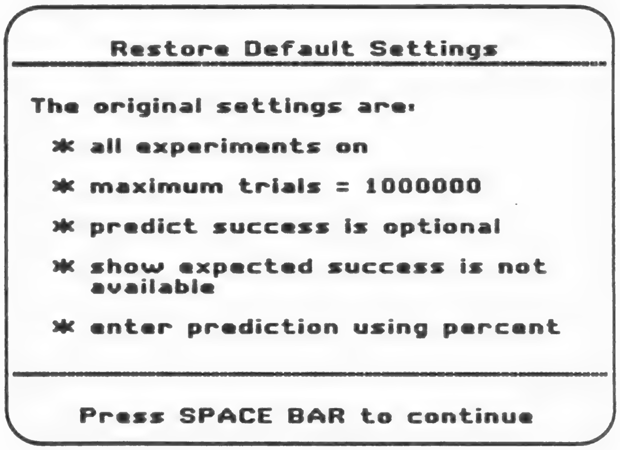
This option will restore the Teacher Options to their original settings.

Are you sure you want to do this?

Yes [✓ No]

Use arrows to move. Press Return.
Escape: Teacher Options ?: Help

Figure 9



Restore Default Settings

The original settings are:

- all experiments on
- maximum trials = 1000000
- predict success is optional
- show expected success is not available
- enter prediction using percent

Press SPACE BAR to continue

Figure 10

Printer Support

If you have a printer attached to your computer, you can print the summary information at the conclusion of each experiment. You are also able to print the first 100 outcomes from any experiment. If your printer does not work properly when trying to print this information, consult pages 56-57.

INTRODUCING STUDENTS TO *PROBABILITY LAB*

After you have familiarized yourself with ***Probability Lab***'s many features, introduce your students to its wide range of capabilities. Begin by choosing an example of an experiment that is easily understood by all (e.g., flipping a single coin a total of 10,000 times and noting the probability of getting a head). Demonstrate how to change the number of coins and number of trials that are used in the experiment. Be sure to have students predict what they think the rate of success will be. When running the actual experiment, start out in the "step mode" and point out how the counters at the top of the screen are keeping track of needed information. Press the ? Key to illustrate the "Status" function that describes the experiment and shows the current rate of success. Then, run the experiment again using two coins and find the probability of getting two heads. Finally, let students work together to design a coin experiment of their own.

The type of modifications you choose to make on the above demonstration will, of course, depend on the type and amount of equipment you have available. If you have a large screen viewing system, you could first demonstrate the features to the entire class. If you have a lab setting, you may want to talk the group through the experiments together and answer questions that may arise.

USING *PROBABILITY LAB* WITH A PURPOSE

Having your students use ***Probability Lab*** on their own just because you happen to be doing a unit on probability will not be very productive. It would be similar to sending students to the science lab and just telling them to do some experiments. While a few may enjoy this freedom, most would not know where to begin! There are, however, many good ways to integrate ***Probability Lab*** into your lesson plans.

1) Conducting probability experiments

One of the primary uses of ***Probability Lab*** is, of course, for conducting experiments. You may have a specific experiment or series of experiments that is helpful in developing a particular probability concept or rule. Textbooks often suggest such experiments as enrichment activities. Conducting these experiments is both fun and important for students; however, only a very limited number of trials is usually possible within a class period and the experiments cannot be very complex. ***Probability Lab*** helps overcome these limitations. Be sure that students have a worksheet that clearly explains the experiment and provides for recording their findings and conclusions. Students will be able to simulate thousands of trials of quite complex situations during a single class period. Examples of worksheets for some specific experiments to develop various concepts have been included on pages 50-53.

2) Checking textbook assignments

Students can check and compare the theoretical probability (textbook solution) of an event with the experimental probability. This could be accomplished by simulating problems found in their textbooks that could be modeled using ***Probability Lab***. By first doing a book problem

experimentally, students can gain a better understanding of the problem since they have visually seen it represented. On the other hand, by doing an experiment after working on the problem in their textbook can serve as a way to check students' work and perhaps clarify their thinking. Either approach has sound instructional value.

3) Designing fair games (or unfair ones)

The topic of "fairness" is often associated with games and probability. A game is considered "fair" if all players have the same chance of winning. It is "unfair" if some players have a distinct advantage. Of course, players with an advantage may argue that such a situation is acceptable! An example of a "fair game" is drawing one card from a standard deck and noting the color. If the card is red, player A wins. If the card is black, player B wins. An example of an "unfair game" would be drawing a single card and noting the face value. If it is a king, player A wins. If it is not a king, player B wins. Player B has a very distinct advantage! Designing fair and unfair games is one way to get students to apply rules of probability to a situation that can be tested. Using ***Probability Lab***, have each student design a game that they consider to be fair and then have them play the game with other students and keep track of their wins. Is the game really fair? The participants will quickly point out any advantages or disadvantages that seem to exist!

A more advanced activity involving "fairness" might be to explore games of chance that pay out different amounts based on the "odds" of a given event happening. This system is based on games being "unfair" and regulating the amount of cash or prizes paid. These games are made attractive by offering large amounts of cash or prizes for winning. A good example of this is the lottery played in many states and countries.

4) Math or science project utility

If your students are given the opportunity to do math or science projects, some may want to explore a topic related to probability or one that requires a probabilistic model. In this case, ***Probability Lab*** could provide students with needed data or results for their project.

5) Sampling populations

Sampling a population in order to make predictions about the total population is an important topic in the study of probability and statistics. This process can be introduced and studied using the **Draw Marbles** experiment. Have someone customize the distribution of the marbles used in the experiment. (Do not let the rest of the class see the actual distribution of marble types.) Provide the class ahead of time with the total number of marbles and the size of the sample. Marbles can then be drawn to satisfy the desired sample size and students can write down their prediction as to what they think the total population will look like. Press the ? Key to reveal the actual makeup of the population.

Experiment Descriptions

General Information 23
Flip Coins 24
Roll Dice 28
Draw Cards 32
Spin Wheels 36
Draw Marbles 40
Pick Numbers 44

GENERAL INFORMATION

The same procedure is used to design and run all of the ***Probability Lab*** experiment types. A few differences, however, do exist in the available settings and ways to define success for each experiment. For example, the physical appearance of coins and cards cannot be modified, but dice can be modified to have different numbers of faces. If you have a basic understanding of the type of experiment you are using, running the experiment should not be difficult.

Do not forget to check out the features available in the Teacher Options found on this disk. Information on accessing and using Teacher Options is found on pages 16-19. These options allow you to adapt the experiments to your particular needs and the needs of your students.

Whenever probability experiments are conducted that require a random number, the validity of the experiment rests on the "randomness" of the random-number generator being used. There are different types of random-number generators that can be used when conducting an experiment, such as random-number tables, phone books (use last four digits of phone numbers), certain mathematical functions, and the physical drawing of objects from a set group. ***Probability Lab*** relies on a machine language routine that is extremely fast, very random, and does the success checking required. The traditional Applesoft BASIC random number function (RND) is not used because it would drastically increase the time required to complete an experiment. An experiment that takes only a few minutes to complete using ***Probability Lab*** would take several hours to complete if done using Applesoft BASIC. For all practical purposes, you should find that the random-number generator employed in ***Probability Lab*** is very random and quite valid.

Some suggested problems for solution and worksheets for investigation are included in this manual. Teachers are encouraged to develop their own problem sets and worksheets to fit their curriculum needs and to match their teaching styles. When students raise questions about "What if?" and no suitable worksheet exists, encourage them to make a table and keep track of their findings in an organized way.

Finally, remember that ***Probability Lab*** is a utility and can serve as an excellent tool to use in your study of probability to test hypotheses, develop concepts, verify principles, and model real-world situations.

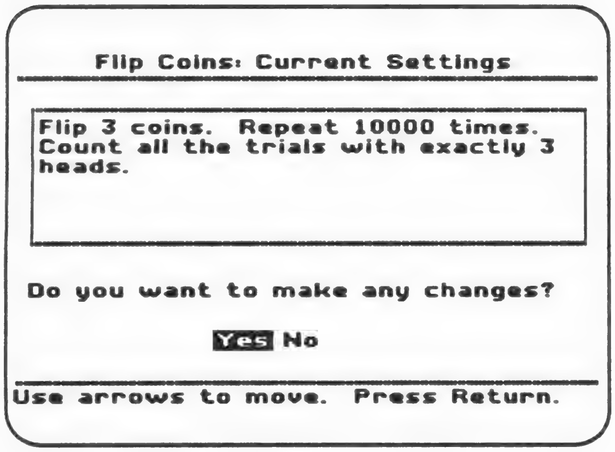
FLIP COINS

Background Information

Flipping coins is the most common type of experiment used in probability. It is also a very common activity used to break ties, to decide who will be first, and, in football, to determine which team kicks off. Because a coin flip can only result in two possible outcomes, it is often referred to as *binomial probability*. The simplicity of coin flipping makes it an excellent way to introduce students to experimental probability. Coins can be used to simulate many real-life experimental situations involving such conditions as yes/no, true/false, male/female, and on/off.

Designing a Coin Experiment

When **Flip Coins** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Coin experiments can use up to 10 coins and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials, using 3 heads as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.

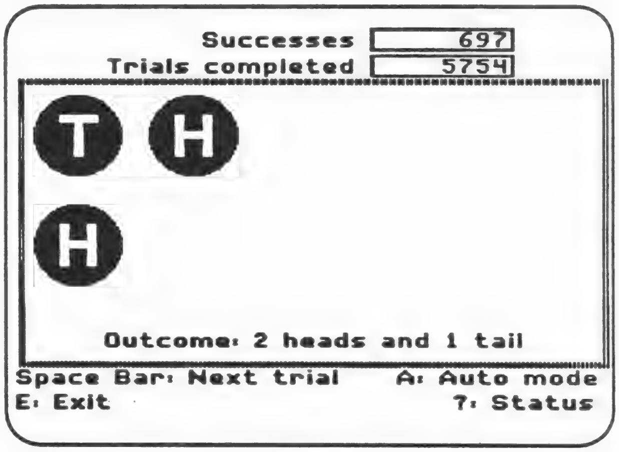


Flip Coins: Current Settings

Flip 3 coins. Repeat 10000 times.
Count all the trials with exactly 3 heads.

Do you want to make any changes?

Figure 1

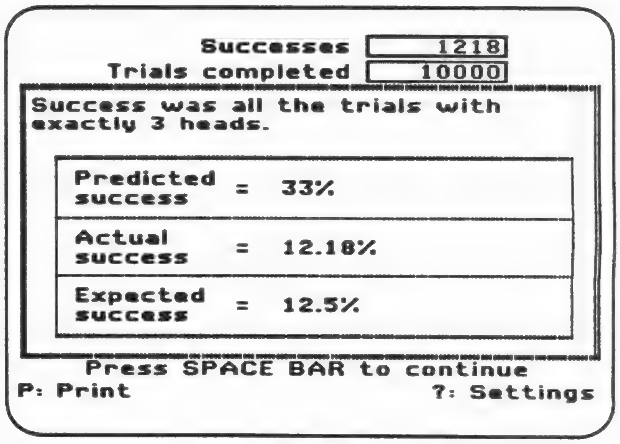


Successes 697
Trials completed 5754

Outcome: 2 heads and 1 tail

Figure 2

At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running any coin experiment are like those used in the experiment illustrated on pages 9-14.



Successes 1218
Trials completed 10000

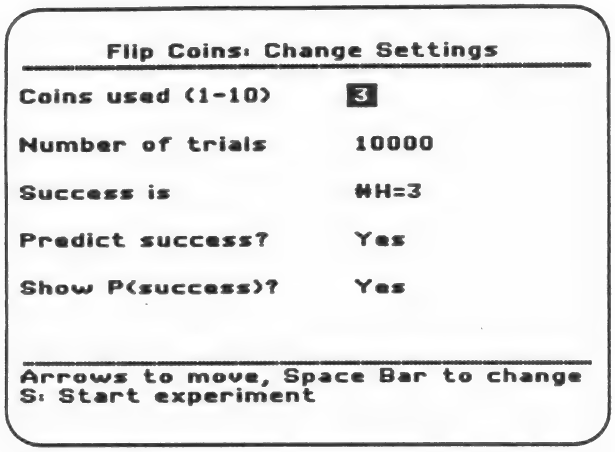
Success was all the trials with exactly 3 heads.

Predicted success = 33%
Actual success = 12.18%
Expected success = 12.5%

Figure 3

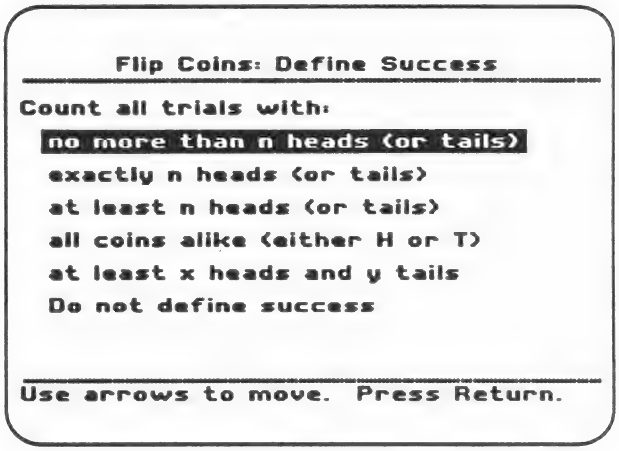
FLIP COINS (continued)

The settings that can be changed in a coin experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



```
        Flip Coins: Change Settings
Coins used (1-10)           3
Number of trials            10000
Success is                  #H=3
Predict success?            Yes
Show P(success)?            Yes

Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4



```
        Flip Coins: Define Success
Count all trials with:
  no more than n heads (or tails)
  exactly n heads (or tails)
  at least n heads (or tails)
  all coins alike (either H or T)
  at least x heads and y tails
  Do not define success

Use arrows to move.  Press Return.
```

Figure 5

Coins used (1 - 10): Up to 10 coins can be flipped in each trial of your experiment.

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 5). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, **#H=3** means "**Count all trials with exactly 3 heads.**" If "**Do not define success**" is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options (see pages 17-18). It allows you to predict the probability of success before running the experiment. At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. The primary method used to calculate the expected success in coin experiments is Pascal's Triangle, which readily gives the possible outcomes for different numbers of coins.

FLIP COINS (continued)

Time Tables for Coin Experiments

The time needed to complete an experiment in "auto mode" depends on the number of trials, the number of coins flipped in each trial, and the computer being used. The definition of success does not influence the time required.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

# of coins	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:02	0:00:10	0:00:20	0:01:37	0:03:13	0:16:02	0:32:04
2	0:00:03	0:00:12	0:00:24	0:01:47	0:03:53	0:19:22	0:38:44
3	0:00:03	0:00:14	0:00:28	0:02:16	0:04:32	0:22:40	0:45:20
4	0:00:04	0:00:16	0:00:32	0:02:38	0:05:15	0:26:13	0:52:26
5	0:00:04	0:00:18	0:00:36	0:02:56	0:05:52	0:29:19	0:58:38
6	0:00:04	0:00:20	0:00:40	0:03:16	0:06:32	0:32:37	1:05:14
7	0:00:05	0:00:22	0:00:44	0:03:36	0:07:12	0:35:57	1:11:54
8	0:00:05	0:00:24	0:00:48	0:03:56	0:07:52	0:39:19	1:18:38
9	0:00:06	0:00:26	0:00:52	0:04:16	0:08:32	0:42:39	1:25:18
10	0:00:06	0:00:28	0:00:56	0:04:38	0:09:15	0:46:12	1:32:24

Approximate Times for an Apple IIGS (fast speed)

# of coins	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:04	0:00:08	0:00:39	0:01:17	0:06:24	0:12:48
2	0:00:01	0:00:05	0:00:10	0:00:48	0:01:35	0:07:54	0:15:48
3	0:00:01	0:00:06	0:00:11	0:00:55	0:01:50	0:09:10	0:18:20
4	0:00:02	0:00:07	0:00:13	0:01:04	0:02:07	0:10:33	0:21:06
5	0:00:02	0:00:07	0:00:15	0:01:11	0:02:22	0:11:48	0:23:36
6	0:00:02	0:00:08	0:00:16	0:01:20	0:02:39	0:13:14	0:26:28
7	0:00:02	0:00:09	0:00:18	0:01:27	0:02:54	0:14:28	0:28:56
8	0:00:02	0:00:10	0:00:19	0:01:35	0:03:10	0:15:48	0:31:36
9	0:00:02	0:00:11	0:00:21	0:01:43	0:03:25	0:17:05	0:34:10
10	0:00:03	0:00:12	0:00:23	0:01:51	0:03:42	0:18:28	0:36:56

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

FLIP COINS (continued)

Some Coin Problems

1. Flip 1 coin a total of 100 times. What percent of the flips were heads? Now flip the coin 1,000 times. What percent of the flips were heads? Repeat for 5,000 flips and 10,000 flips. Do you notice any trend in the rate of success as the number of flips increases?

2. If you flip two coins, what is the probability that both coins will be alike (either both heads or both tails)? Repeat the experiment, using different numbers of coins.

3. Flip 2 coins at a time. Find the probability that a) both coins will be heads, b) both coins will be tails, and c) one coin will be a head and the other a tail. Make a drawing of all the possible ways that 2 coins can land (this is called a sample space). Can you find a relationship between your experimental outcomes and the sample space you constructed?

4. A family has 7 children. What is the probability that: a) exactly 3 of the children are boys? b) all the children are girls? c) there are at least 2 boys and 2 girls?

5. What happens to the probability of getting all heads as the number of coins increases? Start with 1 coin and find the probability of getting a head. Then repeat the experiment using 2 coins and record the probability of getting 2 heads. Repeat this for 3 through 10 coins.

6. Flip an even number of coins (2, 4, 6, 8, and 10) and investigate the probability of getting an equal number of heads and tails as the number of coins increases.

7. Flip 5 coins at a time and find the probability of getting no more than 2 heads ($\#H \leq 2$). Now repeat the experiment and find the probability of getting exactly 3 heads ($\#H = 3$). Finally, repeat the experiment and find the probability of getting at least 4 heads ($\#H \geq 4$). Do you notice anything special about your findings from these three experiments?

8. A student forgot to study for an important test consisting of 10 True/False questions. When taking the test, the student had no idea of what the correct answers were, so each answer was only a guess. In order to pass the test, a score of 7 or more correct was required. What is the probability that the student passed the test?

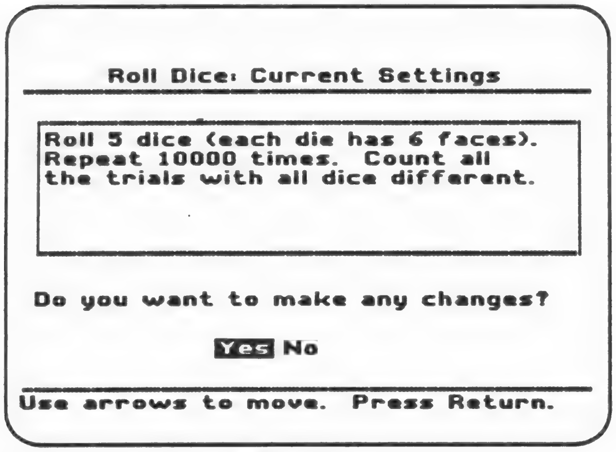
ROLL DICE

Background Information

The standard six-sided die as we know it today has been around for more than 3500 years. It was often used as a device for settling disputes and dividing property. In the Bible, this practice is often referred to as the "casting of lots." Dice were also used in various games of chance, and the science of probability evolved from questions regarding the chances of winning these games. Such noted mathematicians as Galileo and Pascal carefully researched dice games for friends that wanted to discover ways to improve their chances at winning. Today, dice still provide an interesting model for studying probability.

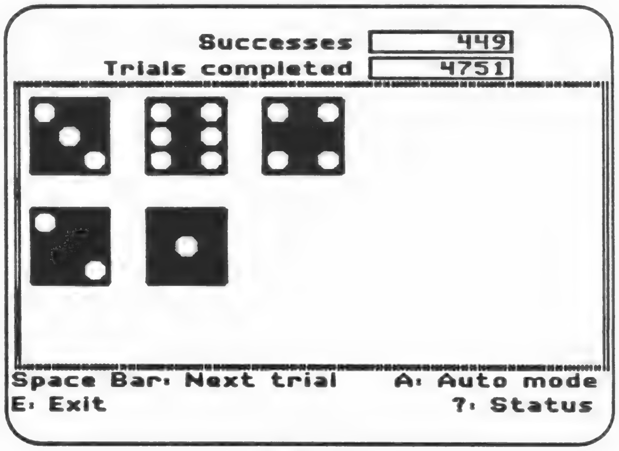
Designing a Dice Experiment

When **Roll Dice** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Dice experiments can use up to 10 dice and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials with all dice different as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.



```
Roll Dice: Current Settings

Roll 5 dice (each die has 6 faces).
Repeat 10000 times.  Count all
the trials with all dice different.

Do you want to make any changes?

Yes No

Use arrows to move.  Press Return.
```

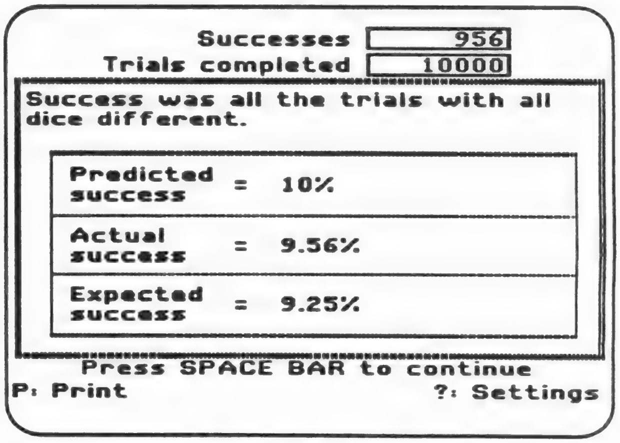
Figure 1



```
      Successes [    449]
Trials completed [   4751]

Space Bar: Next trial    A: Auto mode
E: Exit                     ?: Status
```

Figure 2

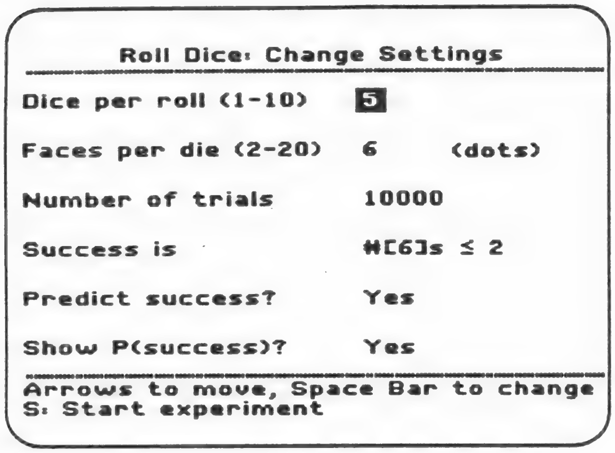
At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running any dice experiment are like those used in the experiment illustrated on pages 9-14.



```
      Successes [    956]
Trials completed [  10000]

Success was all the trials with all
dice different.

   Predicted
   success     =   10%
   Actual
   success     =   9.56%
   Expected
   success     =   9.25%

     Press SPACE BAR to continue
P: Print                 ?: Settings
```

Figure 3

The settings that can be changed in a dice experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



Roll Dice: Change Settings

Dice per roll (1-10) 5

Faces per die (2-20) 6 (dots)

Number of trials 10000

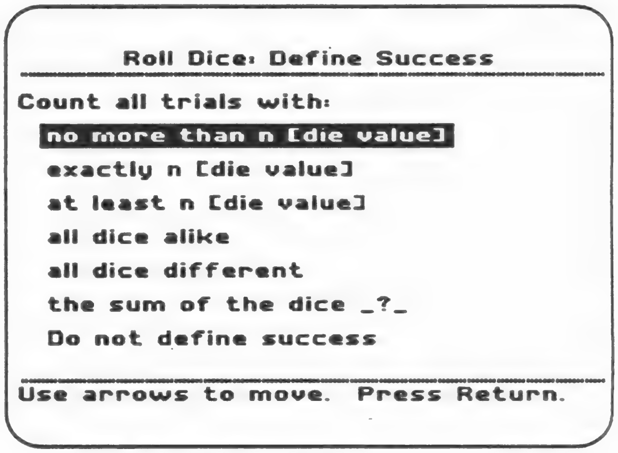
Success is #[6]s ≤ 2

Predict success? Yes

Show P(success)? Yes

Arrows to move, Space Bar to change
S: Start experiment

Figure 4



Roll Dice: Define Success

Count all trials with:

- no more than n [die value]
- exactly n [die value]
- at least n [die value]
- all dice alike
- all dice different
- the sum of the dice _?_
- Do not define success

Use arrows to move. Press Return.

Figure 5

Dice per roll (1 - 10): Up to 10 dice can be rolled in each trial of your experiment.

Faces per die (2 - 20): This setting lets you customize your dice so that each die has from 2 to 20 numbered faces. If you choose 6 faces (standard die), they may be either dotted or numbered.

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 3). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, **#[6]s ≤ 2** means **"Count all trials with no more than 2 [6]s."** If **"Do not define success"** is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options (see pages 17-18). It allows you to predict the probability of success before running the experiment. At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. A variety of recursive counting methods are used in making these calculations.

ROLL DICE (continued)

Time Tables for Dice Experiments

The time needed to run an experiment in "auto mode" depends on the number of trials, the number of dice being rolled in each trial, the type of success checking (using sums increases the times slightly), and the type of computer being used. In the following tables, standard dice with 6 faces and the success check of "exactly 1 [six]" was used.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

# of dice	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:02	0:00:11	0:00:21	0:01:45	0:03:29	0:17:23	0:34:46
2	0:00:03	0:00:13	0:00:26	0:02:08	0:04:16	0:21:20	0:42:40
3	0:00:04	0:00:16	0:00:32	0:02:37	0:05:13	0:26:03	0:52:06
4	0:00:04	0:00:19	0:00:37	0:03:01	0:06:02	0:30:09	1:00:18
5	0:00:05	0:00:21	0:00:42	0:03:28	0:06:55	0:34:33	1:09:06
6	0:00:05	0:00:24	0:00:47	0:03:53	0:07:46	0:38:46	1:17:32
7	0:00:06	0:00:26	0:00:52	0:04:20	0:08:40	0:43:19	1:26:38
8	0:00:06	0:00:29	0:00:57	0:04:45	0:09:29	0:47:25	1:34:50
9	0:00:07	0:00:32	0:01:03	0:05:12	0:10:23	0:51:51	1:43:42
10	0:00:07	0:00:34	0:01:08	0:05:39	0:11:17	0:56:23	1:52:46

Approximate Times for an Apple IIGS (fast speed)

# of dice	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:05	0:00:09	0:00:41	0:01:22	0:06:50	0:13:40
2	0:00:01	0:00:06	0:00:11	0:00:52	0:01:43	0:08:35	0:17:10
3	0:00:02	0:00:07	0:00:13	0:01:03	0:02:06	0:10:29	0:20:58
4	0:00:02	0:00:08	0:00:15	0:01:13	0:02:26	0:12:08	0:24:16
5	0:00:02	0:00:09	0:00:17	0:01:24	0:02:48	0:13:56	0:27:52
6	0:00:02	0:00:10	0:00:19	0:01:34	0:03:08	0:15:36	0:31:12
7	0:00:02	0:00:11	0:00:21	0:01:45	0:03:30	0:17:26	0:34:52
8	0:00:03	0:00:12	0:00:23	0:01:55	0:03:49	0:19:05	0:38:10
9	0:00:03	0:00:13	0:00:25	0:02:06	0:04:11	0:20:52	0:41:44
10	0:00:03	0:00:14	0:00:28	0:02:17	0:04:33	0:22:43	0:45:26

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

ROLL DICE (continued)

Some Dice Problems

1. Roll a single six-sided die several times and note the probability of getting a [1]. Repeat the experiment for each of the other face values [2], [3], [4], [5], and [6]. How do the outcomes compare?

2. Roll 2 six-sided dice and note the sum of each trial. Sums will range from 2 to 12. Are the chances of getting a sum of 2 any different than getting a sum of 7?

3. Roll 2 six-sided dice and note the sum of each trial. Are the chances of getting an odd-numbered sum any different than the chances of getting an even-numbered sum? Why or why not?

4. Roll 10 twenty-sided dice and note the sum of each trial. What is the probability of getting the lowest possible sum (10)? Of getting the highest possible sum (200)? What possible sum seems to be the most frequently occurring sum?

5. Roll 2 six-sided dice and note the sum on each trial. Can you design a series of experiments that will allow you to find the probability of getting a sum between 4 and 7 ($4 \leq \text{sum} \leq 7$)?

6. There are 7 people sitting at a table. Assuming that the chances of being born in any given month of the year are equal, what is the probability that everyone within this group of people was born in a different month? That everyone was born in the same month? That at least 3 of the people were born in January? That exactly 1 person was born in December? That at least 3 people shared the same birth month?

7. A test consists of 10 multiple choice questions with 4 choices for each question. What is your chance of passing such a test (getting 6 or more questions correct)?

8. Roll 5 dice at a time and note the sum of the dice. How many faces should the dice have in order to maximize your chances of getting a sum of 25?

9. A marketing promotion involves giving away a set of 6 baseball cards featuring 6 different All-Star players. One card is placed in each box of Super Duper Corn Flakes. Assume that an equal number of cards were randomly placed in the boxes. How many boxes of cereal will probably have to be purchased in order to obtain a complete set of the 6 different baseball cards? (Hint: Use the "step mode" with a single six-sided die and check off the cards one at a time as you obtain them. Once you have the complete set, note the number of trials that it took. Repeat the experiment several times and find the average number of boxes required.)

10. The promotion in problem 9 was so successful that Super Duper Corn Flakes decided to repeat the promotion with a set of 12 different baseball cards. How many boxes will it probably take to complete your set?

11. A popular casino game called Chuck-a-Luck is played with 3 six-sided dice. A player bets a dollar (or some other amount) on one of the face values (1-6) appearing. As an example, assume that you bet \$1 on the number 3 appearing. If one of the dice is a 3, you win \$1. If two of the dice are 3's, you win \$2. If all three of the dice are 3's, you win \$3. Investigate the probabilities of losing your \$1, winning \$1, winning \$2, and winning \$3. Does this seem to be a fair game? If not, does the game favor you or the casino?

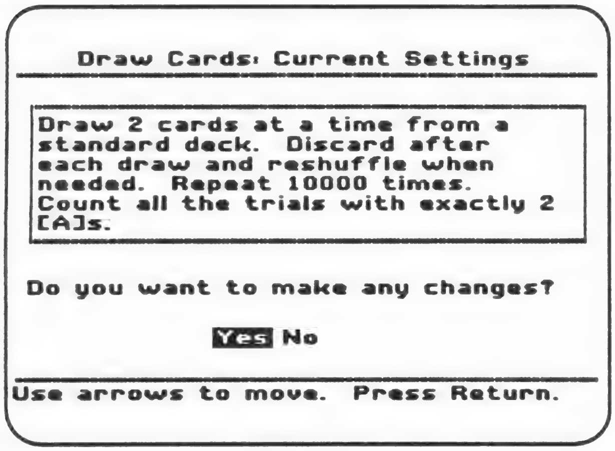
DRAW CARDS

Background Information

The exact origin of cards is not known, but evidence suggests that cards may have been developed in China in about the seventh century. Some of the earliest forms of cards were called Tarot cards and probably originated in Italy. The modern 52-card deck that has become the standard throughout the world was developed in France. Through the centuries, many different card games have been invented and it remains a very popular type of game. Because cards are so popular and the probability of winning various card games has been carefully studied, they are often used in the study of probability.

Designing a Card Experiment

When **Draw Cards** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Card experiments can draw up to 10 cards and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials with exactly 2 aces as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.

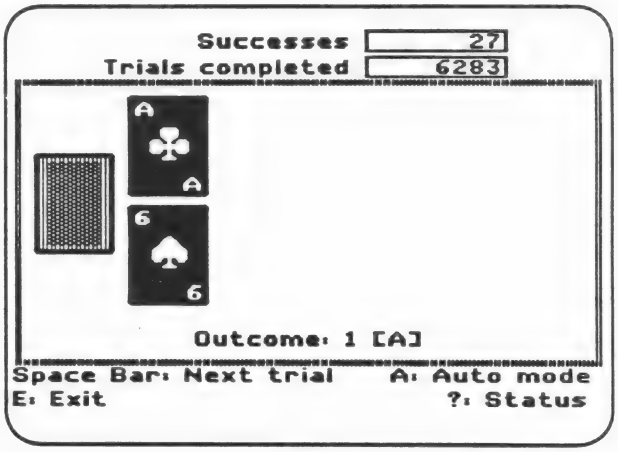


Draw Cards: Current Settings

Draw 2 cards at a time from a standard deck. Discard after each draw and reshuffle when needed. Repeat 10000 times. Count all the trials with exactly 2 [A]s.

Do you want to make any changes?

Figure 1



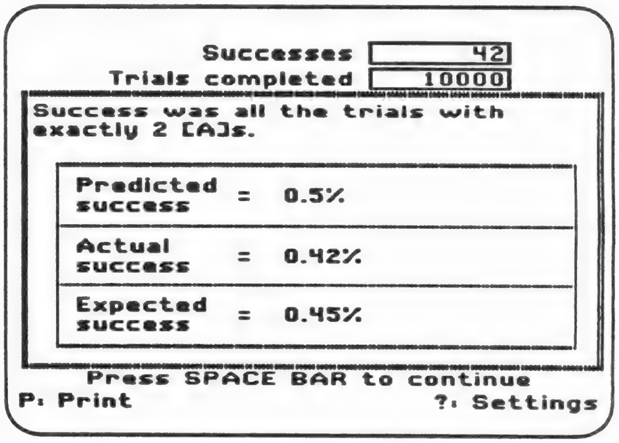
Successes 27

Trials completed 6283

Outcome: 1 [A]

Figure 2

At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running any card experiment are like those used in the experiment illustrated on pages 9-14.



Successes 42

Trials completed 10000

Success was all the trials with exactly 2 [A]s.

Predicted success = 0.5%

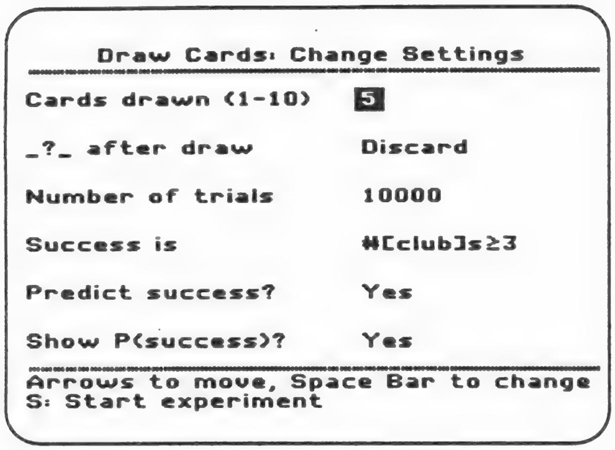
Actual success = 0.42%

Expected success = 0.45%

Figure 3

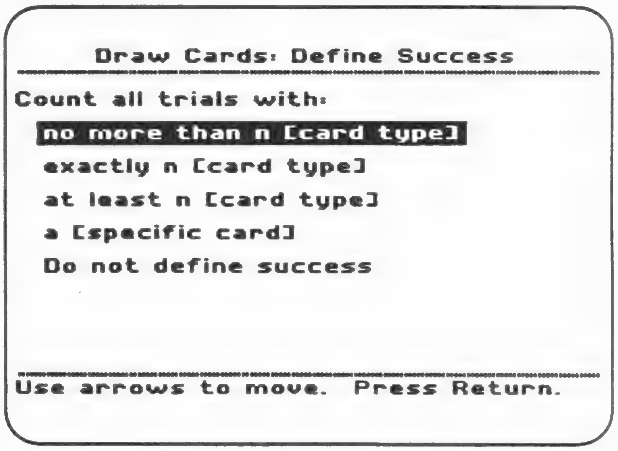
DRAW CARDS (continued)

The settings that can be changed in a card experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



```
        Draw Cards: Change Settings
Cards drawn (1-10)      5
_?_ after draw          Discard
Number of trials        10000
Success is              #[club]s≥3
Predict success?        Yes
Show P(success)?        Yes
Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4



```
        Draw Cards: Define Success
Count all trials with:
 no more than n [card type]
 exactly n [card type]
 at least n [card type]
 a [specific card]
 Do not define success
Use arrows to move.  Press Return.
```

Figure 5

Cards drawn (1 - 10): Up to 10 cards can be drawn in each trial of your experiment.

_?_ after draw: This option can be set to "Reshuffle" (cards are replaced and the deck is shuffled between draws) or to "Discard" (cards are discarded after each draw and the deck is reshuffled when there are not enough cards remaining to make another draw).

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 5). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, **#[club]s ≥ 3** means **"Count all trials with at least 3 clubs."** If **"Do not define success"** is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options. It allows one to predict the probability of success before running the experiment (see pages 17-18). At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. The hypergeometric formula is used in most cases to make this calculation.

DRAW CARDS (continued)

Time Tables for Card Experiments

The time needed to complete an experiment in "auto mode" depends on the number of trials, the number of cards per draw, whether or not the cards are reshuffled after each draw, and the type of computer being used. Discarding after each draw results in faster times since the deck is reshuffled only when necessary. The tables below are for experiments with discarding.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

Cards drawn	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:03	0:00:12	0:00:23	0:01:53	0:03:45	0:18:42	0:37:24
2	0:00:03	0:00:15	0:00:30	0:02:26	0:04:51	0:24:12	0:48:24
3	0:00:04	0:00:19	0:00:37	0:03:02	0:06:04	0:30:17	1:00:34
4	0:00:05	0:00:22	0:00:43	0:03:32	0:07:04	0:35:18	1:10:36
5	0:00:05	0:00:26	0:00:51	0:04:12	0:08:24	0:41:57	1:23:54
6	0:00:06	0:00:30	0:00:59	0:04:55	0:09:50	0:49:06	1:38:12
7	0:00:07	0:00:33	0:01:06	0:05:26	0:10:52	0:54:17	1:48:34
8	0:00:08	0:00:37	0:01:14	0:06:10	0:12:19	1:01:31	2:03:02
9	0:00:09	0:00:43	0:01:25	0:07:06	0:14:11	1:10:53	2:21:46
10	0:00:09	0:00:43	0:01:25	0:07:06	0:14:11	1:10:53	2:21:46

Approximate Times for an Apple IIGS (fast speed)

Cards drawn	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:05	0:00:10	0:00:46	0:01:31	0:07:35	0:15:10
2	0:00:02	0:00:06	0:00:12	0:00:59	0:01:58	0:09:49	0:19:38
3	0:00:02	0:00:08	0:00:15	0:01:14	0:02:28	0:12:17	0:24:34
4	0:00:02	0:00:09	0:00:18	0:01:26	0:02:52	0:14:19	0:28:38
5	0:00:02	0:00:11	0:00:21	0:01:43	0:03:25	0:17:01	0:34:02
6	0:00:03	0:00:12	0:00:24	0:02:00	0:03:59	0:19:55	0:39:55
7	0:00:03	0:00:14	0:00:27	0:02:13	0:04:25	0:22:01	0:44:02
8	0:00:03	0:00:15	0:00:30	0:02:30	0:05:00	0:24:57	0:49:54
9	0:00:04	0:00:18	0:00:35	0:02:53	0:05:45	0:28:45	0:57:30
10	0:00:04	0:00:18	0:00:35	0:02:53	0:05:45	0:28:45	0:57:30

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

Some Card Problems

1. Draw 1 card from a standard deck. What is the probability that the card drawn is red? That it is a diamond? That it is a king? That it is the queen of spades?
2. Draw 5 cards from a standard deck. What is the probability that the hand drawn will contain 4 aces?
3. Draw 5 cards from a standard deck. What is the probability of drawing a heart flush (all 5 cards are hearts)?
4. Draw 1 card from a standard deck. What is the probability that it is not an ace?
5. Draw 5 cards from a standard deck. What is the probability that the hand does not contain an ace?

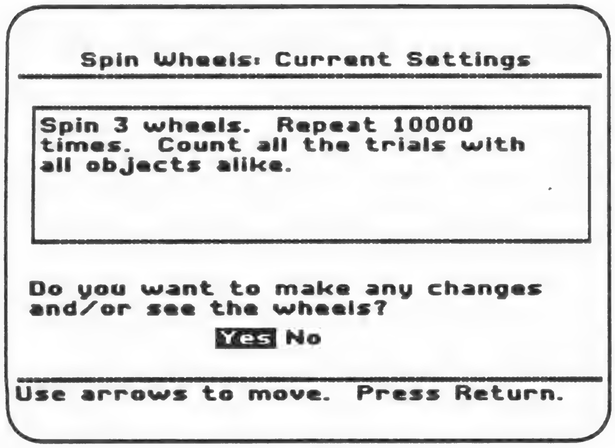
SPIN WHEELS

Background Information

Many forms of gambling are based on devices that feature spinning wheels that are spun and then stop on one item on the wheel. Examples of spinning wheels include the roulette wheel, slot machines, and the wheel of fortune. The wheels generally contain unequal amounts of different types of objects—pictures, numbers, and colors—and when a number of wheels are used together, certain combinations occur very infrequently. This is the basis for the modern-day slot machine. These machines usually have from 3 to 6 wheels and pay huge jackpots when all the wheels, once stopped, show a certain combination of items. Of course, the probability for certain combinations occurring is known and taken into consideration in determining the amounts paid.

Designing a Wheel Experiment

When **Spin Wheels** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Wheel experiments can use up to 5 wheels and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials with all objects alike as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.

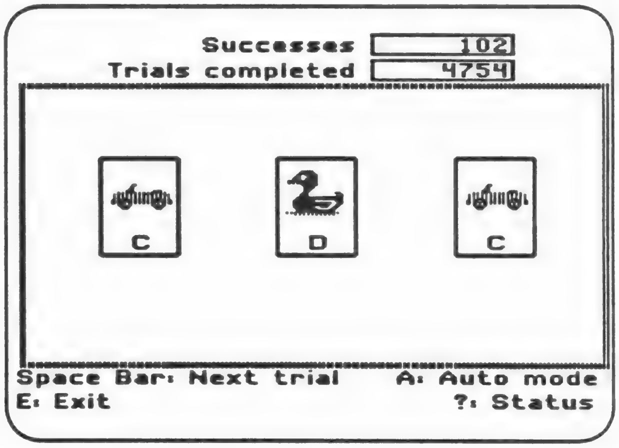


Spin Wheels: Current Settings

Spin 3 wheels. Repeat 10000 times. Count all the trials with all objects alike.

Do you want to make any changes and/or see the wheels?

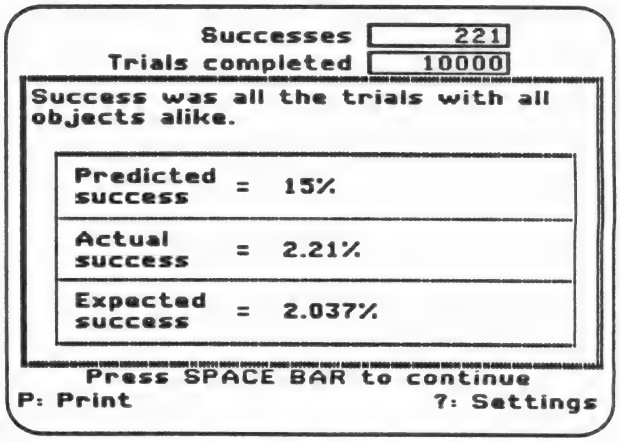
Figure 1



Successes 102
Trials completed 4754

Figure 2

At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running any wheel experiment are like those used in the experiment illustrated on pages 9-14.



Successes 221
Trials completed 10000

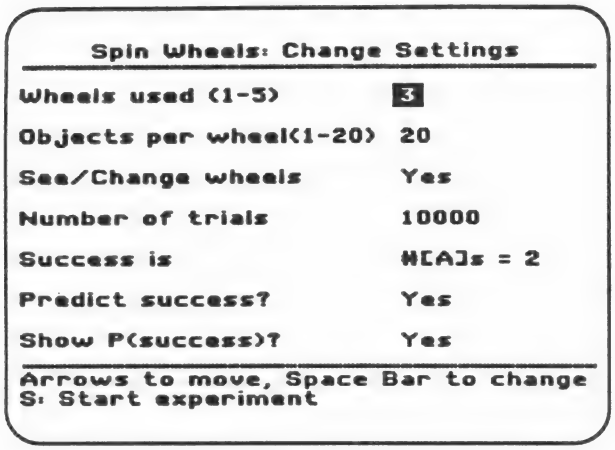
Success was all the trials with all objects alike.

Predicted success = 15%
Actual success = 2.21%
Expected success = 2.037%

Figure 3

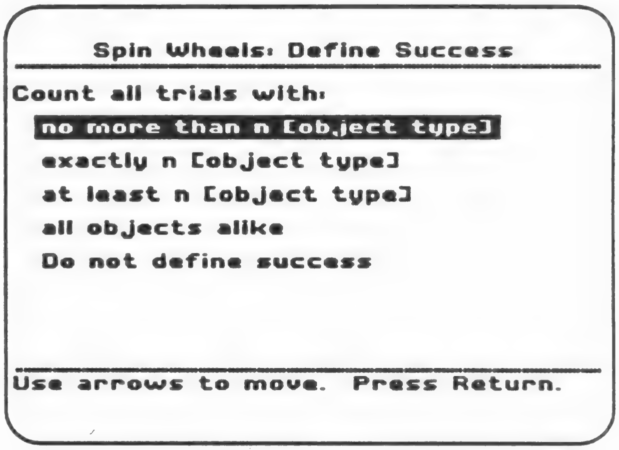
SPIN WHEELS (continued)

The settings that can be changed in a wheel experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



```
      Spin Wheels: Change Settings
Wheels used (1-5)          3
Objects per wheel(1-20)    20
See/Change wheels          Yes
Number of trials           10000
Success is                 #[A]s = 2
Predict success?           Yes
Show P(success)?           Yes
Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4



```
      Spin Wheels: Define Success
Count all trials with:
  no more than n [object type]
  exactly n [object type]
  at least n [object type]
  all objects alike
  Do not define success

Use arrows to move.  Press Return.
```

Figure 5

Wheels used (1 - 5): You can spin up to 5 wheels at a time in each trial of your experiment.

Objects per wheel (1 - 20): You can place up to 20 objects on each wheel. There are six different objects to choose from—[A]pples, [B]oats, [C]ars, [D]ucks, [E]lephants, and [F]lowers.

See/Change wheels: This option allows you to see the current design of the wheels. If you would like a different distribution of objects, you can redesign the wheels yourself or let the computer do it.

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 5). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, **#[A]s = 2** means **"Count all trials with exactly 2 [A]pples."** If **"Do not define success"** is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options (see pages 17-18). It allows you to predict the probability of success before running the experiment. At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. Most of these calculations are made by multiplying the probabilities of a given event happening (or not happening) on each wheel and counting recursively all the possibilities.

SPIN WHEELS (continued)

Time Tables for Wheel Experiments

The time required to complete an experiment running in "auto mode" will depend on the number of trials, the number of wheels being used, and the type of computer used.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

# of wheels	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:03	0:00:14	0:00:27	0:02:11	0:04:21	0:21:45	0:43:30
2	0:00:04	0:00:19	0:00:37	0:03:03	0:06:06	0:30:26	1:00:52
3	0:00:05	0:00:24	0:00:47	0:03:56	0:07:51	0:39:13	1:18:26
4	0:00:06	0:00:29	0:00:58	0:04:48	0:09:35	0:47:55	1:35:50
5	0:00:07	0:00:35	0:01:09	0:05:42	0:11:24	0:56:57	1:53:54

Approximate Times for an Apple IIGS (fast speed)

# of wheels	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:06	0:00:11	0:00:53	0:01:45	0:08:45	0:17:30
2	0:00:02	0:00:08	0:00:15	0:01:14	0:02:27	0:12:15	0:24:30
3	0:00:02	0:00:10	0:00:19	0:01:35	0:03:10	0:15:47	0:31:34
4	0:00:03	0:00:12	0:00:24	0:01:56	0:03:52	0:19:17	0:38:34
5	0:00:03	0:00:14	0:00:28	0:02:18	0:04:35	0:22:55	0:45:50

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

Some Wheel Problems

1. A single wheel contains 20 objects (5 [A]pples, 5 [B]oats, 5 [C]ars, and 5 [D]ucks). What is the probability of getting an [A]pple? Of getting a [B]oat? Of getting a [C]ar? Of getting a [D]uck?

2. When you first access the Pick Wheels experiment, the default setting is set to simulate a standard three-wheel slot machine. What are the chances of getting 3 [A]pples? Of getting 3 [B]oats? Of getting 3 [C]ars? Of getting 3 [D]ucks? Of getting 3 [E]lephants? Of getting 3 [F]lowers?

3. Have someone design a single wheel containing 5 objects so you are not aware of what how many of each type of object are on the wheel. After running an experiment of your design, guess the makeup of the wheel. If incorrect, redesign the experiment and test again. Continue until you have correctly determined the exact makeup of the wheel. How many experiments did it take to get the right answer? Are there some tests that are better than others to help determine the makeup of the wheel? This game can be played with different numbers of objects.

4. The weather report for the weekend says that the probability of rain on Friday is 20%, Saturday 30%, and Sunday 50%. Assuming that all these events are independent of each other, what is the probability that it will rain on all three days? On none of the days? On at least 2 of the days? (Hint: Design a wheel for each day with an appropriate amount of two different types of objects to represent "rain" and "no rain.")

5. You are in a single-elimination arm wrestling tournament along with 15 other people. The 8 first-round winners advance to the second round. The 4 winners in the second round advance to the third round. The 2 winners in the third round wrestle for the championship. If all 16 contestants are all evenly matched and have the same chance of winning, what is the probability that you will win the tournament? (Hint: Use 4 wheels with 16 divisions per wheel. Select one type of object to represent yourself and another type of object to represent the other contestants. Place the appropriate number of objects on each wheel to reflect your chances of winning at each round if you were to advance. Winning the tournament would mean having all 4 wheels stop on the object that represents you.)

6. There is a stretch of highway through Randomsville that has 5 traffic lights. These traffic lights are set so that they are red 50% of the time, green 45% of the time, and yellow 5% of the time. The lights do not work together so you never knows what to expect when approaching a light. What is the probability of driving straight through Randomsville encountering only green lights ? Encountering all red lights? Encountering all yellow lights?

7. Dr. Dunk of the Hoopsville Hoopsters basketball team makes 65% of his free throws. During a recent game, he attempted 5 free throws. What is the probability that he made all 5 free throws? Missed all 5 free throws? Made exactly 3 out of the 5 free throws? Made at least 3 out of the 5 free throws?

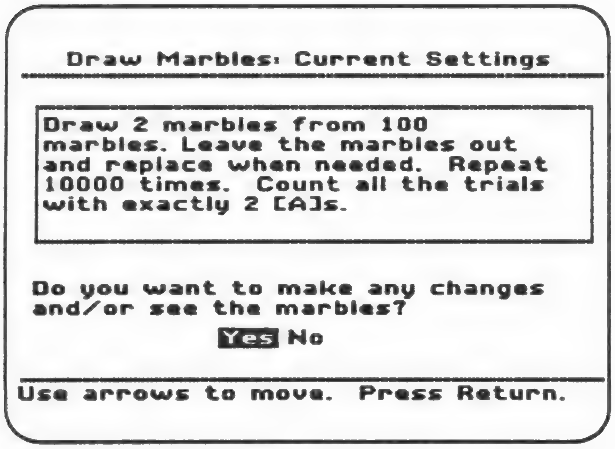
DRAW MARBLES

Background Information

While drawing marbles is not the basis for any popular games, it does provide interesting models for studying probability. Different quantities and colors of marbles can be placed in a bag and then drawn out. The "sample space" for such an experiment is easy to visualize and most students readily understand that the chances are better to draw out some colors than others. This type of experiment allows you to experiment with events that have an unequal probability of happening.

Designing a Marble Experiment

When **Draw Marbles** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Marble experiments can draw up to 10 marbles and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials with exactly 2 [A]s as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.

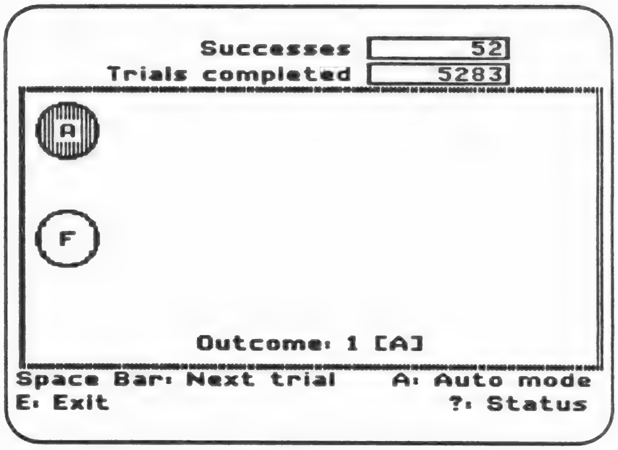


Draw Marbles: Current Settings

Draw 2 marbles from 100 marbles. Leave the marbles out and replace when needed. Repeat 10000 times. Count all the trials with exactly 2 [A]s.

Do you want to make any changes and/or see the marbles?

Figure 1

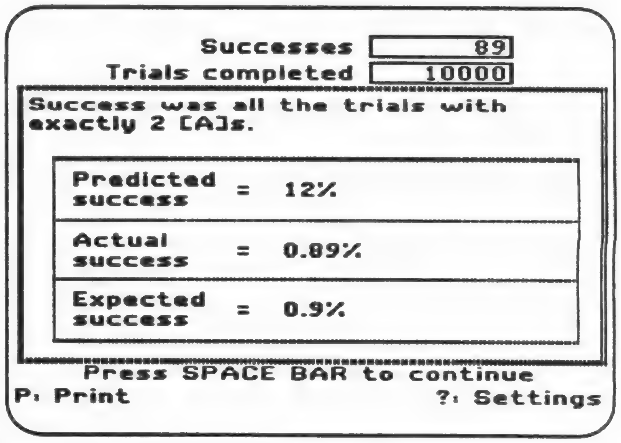


Successes 52
Trials completed 5283

Outcome: 1 [A]

Figure 2

At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running any marble experiment are like those used in the experiment illustrated on pages 9-14.



Successes 89
Trials completed 10000

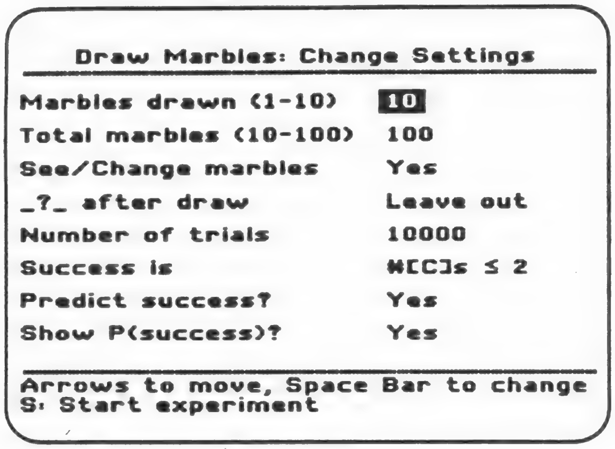
Success was all the trials with exactly 2 [A]s.

Predicted success = 12%
Actual success = 0.89%
Expected success = 0.9%

Figure 3

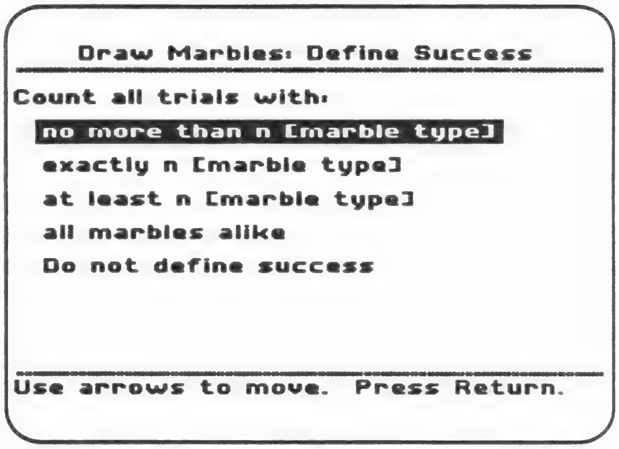
DRAW MARBLES (continued)

The settings that can be changed in a marble experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



```
     Draw Marbles: Change Settings
Marbles drawn (1-10)       10
Total marbles (10-100)     100
See/Change marbles         Yes
_?_ after draw             Leave out
Number of trials           10000
Success is                 #[C]s ≤ 2
Predict success?           Yes
Show P(success)?           Yes
Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4



```
     Draw Marbles: Define Success
Count all trials with:
  no more than n [marble type]
  exactly n [marble type]
  at least n [marble type]
  all marbles alike
  Do not define success
Use arrows to move.  Press Return.
```

Figure 5

Marbles drawn (1 - 10): Up to 10 marbles can be drawn in each trial of your experiment.

Total marbles (10 - 100): Between 10 and 100 total marbles can be used. There are six colors (types) of marbles—[A] violet, [B] green, [C] white, [D] orange, [E] blue, and [F] black.

See/Change marbles: This option allows you to see the current distribution of marbles. If you are not satisfied with the current setup, you can redistribute the marbles yourself or let the computer do it.

—?—after draw: Marbles can be "left out" or "put back" in the bag after each draw.

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 5). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, #[C]s ≤ 2 means **"Count all trials with no more than 2 [C]s (white marbles)."** If **"Do not define success"** is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options (see pages 17-18). It allows one to predict the probability of success before running the experiment. At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. Most of these calculations are based on the use of the hypergeometric formula.

Time Tables for Marble Experiments

The time needed to complete an experiment in "auto mode" depends on the number of trials, the number of marbles per draw, whether or not you are putting marbles back after each draw, and the type of computer used. Leaving marbles out after each draw is much faster because the marbles do not have to be mixed after each draw. The tables below are for experiments in which the marbles are left out after each draw.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

Marbles drawn	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:03	0:00:11	0:00:22	0:01:49	0:03:38	0:18:07	0:36:14
2	0:00:03	0:00:14	0:00:28	0:02:21	0:04:41	0:23:21	0:46:42
3	0:00:04	0:00:18	0:00:35	0:02:52	0:05:43	0:28:33	0:57:06
4	0:00:04	0:00:21	0:00:41	0:03:22	0:06:44	0:33:40	1:07:20
5	0:00:05	0:00:24	0:00:47	0:03:52	0:07:44	0:38:37	1:17:14
6	0:00:06	0:00:27	0:00:54	0:04:30	0:08:59	0:44:55	1:29:50
7	0:00:06	0:00:30	0:01:00	0:04:59	0:09:57	0:49:42	1:39:24
8	0:00:07	0:00:34	0:01:07	0:05:33	0:11:06	0:55:29	1:50:58
9	0:00:08	0:00:36	0:01:12	0:05:57	0:11:54	0:59:26	1:58:52
10	0:00:08	0:00:39	0:01:18	0:06:27	0:12:54	1:04:26	2:08:52

Approximate Times for an Apple IIGS (fast speed)

Marbles drawn	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:05	0:00:09	0:00:44	0:01:28	0:07:19	0:14:38
2	0:00:02	0:00:06	0:00:12	0:00:57	0:01:54	0:09:26	0:18:52
3	0:00:02	0:00:07	0:00:14	0:01:10	0:02:19	0:11:32	0:23:04
4	0:00:02	0:00:09	0:00:17	0:01:22	0:02:44	0:13:36	0:27:12
5	0:00:02	0:00:10	0:00:19	0:01:34	0:03:08	0:15:36	0:31:12
6	0:00:03	0:00:11	0:00:22	0:01:49	0:03:38	0:18:09	0:36:18
7	0:00:03	0:00:12	0:00:25	0:02:01	0:04:01	0:20:05	0:40:10
8	0:00:03	0:00:14	0:00:27	0:02:15	0:04:29	0:22:25	0:44:50
9	0:00:03	0:00:15	0:00:29	0:02:25	0:04:49	0:24:01	0:48:02
10	0:00:04	0:00:16	0:00:32	0:02:37	0:05:13	0:26:02	0:52:04

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

DRAW MARBLES (continued)

Some Marble Problems

1 A bag contains 50 violet marbles (type A) and 50 green marbles (type B). What are the chances of drawing out 2 marbles, both of which are violet? Both green? One of each color?

2. A bag contains 100 marbles. There are exactly 20 marbles of each of 5 different colors. What are the chances of drawing out 2 marbles, both the same color? Of drawing out 2 marbles, each a different color?

3. A race has 25 cars built by four different manufacturers. There are 10 Fords, 8 Chevrolets, 5 Buicks, and 2 Plymouths. Assuming that all the cars have an equal chance of winning, what is the probability that the first three cars to finish the race will all be Buicks? All Chevrolets? All Fords? All Plymouths?

4. The Iceville Icers hockey team has won 54% of their games, lost 40% of their games, and tied 6% of their games. Assuming that they continue to play at the same level, what are the chances that they will win their next 5 games in a row? That they will lose their next 5 games in a row? That they will tie their next 5 games in a row? That they will win at least 2 of their next 5 games?

5. The city of Videoville has 3 television stations. Thirty-nine percent of the population normally watches channel 3 (KWOW) evening news, 32% usually watches channel 7 (KOOL) evening news, 23% watches channel 11 (KXYZ) evening news, and the remaining 6% does not watch the evening news. During the ratings week, a sample poll is taken by randomly calling 10 people in Videoville during the evening news and asking which station they are watching. What is the probability that the sampling will find all 10 people watching channel 3 (KWOW)? Take 10 samplings of 10 calls each. Do the samples seem to reflect the known percentages that watch the various channels?

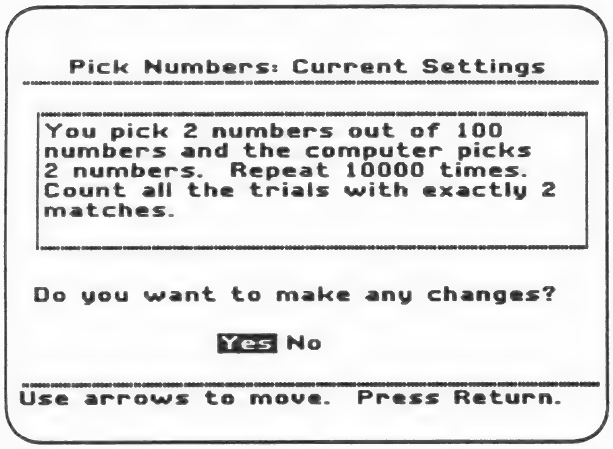
PICK NUMBERS

Background Information

For thousands of years picking numbers has been used as the basis for many games and activities. The simplest example would be picking a number between 1 and 10 to see who will have the first turn. Many games, including Lotto, Bingo, Keno, and state and national lotteries, are based on the principle of matching numbers. Today, lotteries are common throughout the world and take in millions of dollars each year, with about half of it returned to the bettors. Modeling and simulating a lottery is not an easy task. For this reason, it is not often used when studying probability. ***Probability Lab***, however, simplifies this task immensely.

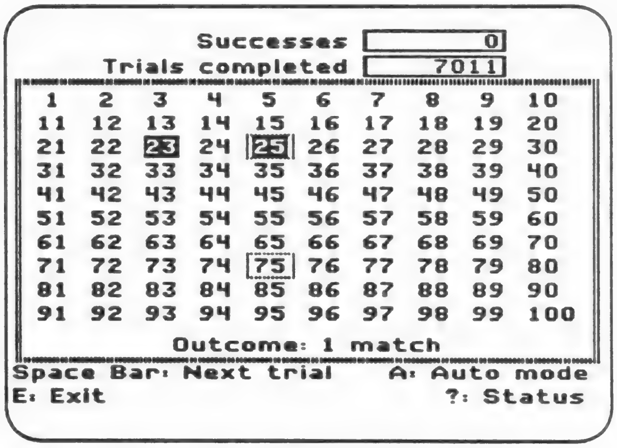
Designing a Number Experiment

When **Pick Numbers** is selected on the main menu, the settings will default to those shown in Figure 1. To make changes, respond with a "Yes." Number experiments can pick up to 10 numbers and success can be defined in several ways. The example shown in Figure 2 is keeping track of all trials with exactly 2 matches as successes. The counters at the top of the screen keep track of the number of successes and trials completed. An experiment can have up to 1,000,000 trials.



```
Pick Numbers: Current Settings

You pick 2 numbers out of 100
numbers and the computer picks
2 numbers.  Repeat 10000 times.
Count all the trials with exactly 2
matches.

Do you want to make any changes?

        Yes No

Use arrows to move.  Press Return.
```

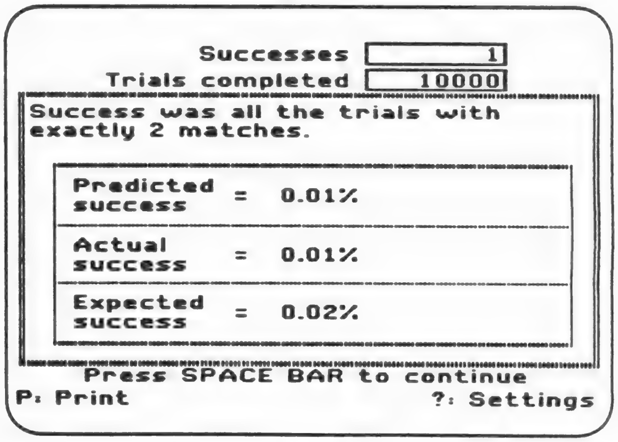
Figure 1



```
           Successes [      0]
     Trials completed [   7011]
 1   2   3   4   5   6   7   8   9   10
11  12  13  14  15  16  17  18  19   20
21  22  23  24  25  26  27  28  29   30
31  32  33  34  35  36  37  38  39   40
41  42  43  44  45  46  47  48  49   50
51  52  53  54  55  56  57  58  59   60
61  62  63  64  65  66  67  68  69   70
71  72  73  74  75  76  77  78  79   80
81  82  83  84  85  86  87  88  89   90
91  92  93  94  95  96  97  98  99  100
          Outcome: 1 match
Space Bar: Next trial     A: Auto mode
E: Exit                      ?: Status
```

Figure 2

At the conclusion of an experiment, the actual success is printed. If the options to predict success and show expected success are on, these are also displayed (Figure 3). The steps in running a numbers experiment are like those used in the experiment illustrated on pages 9-14.

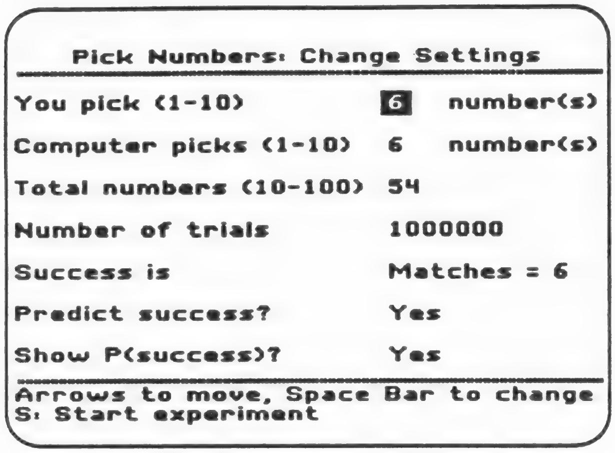


```
           Successes [      1]
     Trials completed [  10000]
Success was all the trials with
exactly 2 matches.

   Predicted   =   0.01%
   success
   Actual      =   0.01%
   success
   Expected    =   0.02%
   success

    Press SPACE BAR to continue
P: Print                 ?: Settings
```

Figure 3

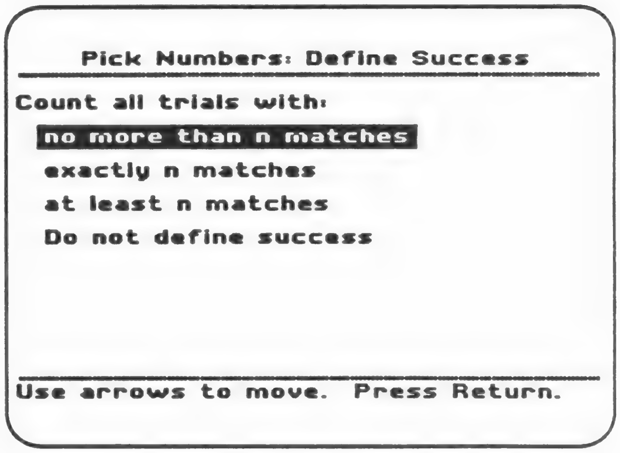
PICK NUMBERS (continued)

The settings that can be changed in a numbers experiment are shown in Figure 4. When necessary, instructions and prompts will be given to guide you in making changes.



```
        Pick Numbers: Change Settings
You pick (1-10)             6   number(s)
Computer picks (1-10)       6   number(s)
Total numbers (10-100)      54
Number of trials            1000000
Success is                  Matches = 6
Predict success?            Yes
Show P(success)?            Yes
Arrows to move, Space Bar to change
S: Start experiment
```

Figure 4



```
       Pick Numbers: Define Success
Count all trials with:
  no more than n matches
  exactly n matches
  at least n matches
  Do not define success

Use arrows to move.  Press Return.
```

Figure 5

You pick (1 - 10): You can pick up to 10 different numbers to be used throughout the experiment. The size of the numbers picked cannot exceed the total numbers used.

Computer picks (1 - 10): This indicates how many numbers are randomly picked by the computer on each trial.

Total numbers (10 - 100): This setting determines the range of numbers available from 1 to the value indicated.

Number of trials: When this number is reached, the experiment will end. This number (maximum of 1,000,000) can be limited using Teacher Options (see page 17).

Success is: This setting defines a successful trial (Figure 5). Prompts and instructions will take you through the selection process. A mathematical expression representing your selection is shown on the "Change Settings" screen. In the screens above, **Matches = 6** means **"Count all trials with exactly 6 matches."** If **"Do not define success"** is selected, you must do your own check for success after each trial.

Predict success?: This option appears when designated as a "student option" in Teacher Options (see pages 17-18). It allows one to predict the probability of success before running the experiment. At the end of the experiment, the prediction is then printed along with the actual rate of success.

Show P(success)?: This option appears when designated as a "student option" in Teacher Options (see page 18). When this option is exercised, the expected (theoretical) probability of success is shown at the end of the experiment. Most of these calculations are based on the use of the hypergeometric formula.

PICK NUMBERS (continued)

Time Tables for Number Experiments

The time needed to complete an experiment in "auto mode" depends on the number of trials, how many numbers are picked by you and the computer, and the computer being used. In the following tables, 100 numbers were used and the same "# of numbers" were used for both the "person picks" and the computer picks.

Approximate Times for an Apple //e, Apple IIc, or Apple IIGS (normal speed)

# of numbers	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:03	0:00:13	0:00:27	0:02:11	0:04:22	0:21:47	0:43:34
2	0:00:04	0:00:18	0:00:36	0:03:00	0:05:59	0:29:55	0:59:50
3	0:00:05	0:00:24	0:00:47	0:03:52	0:07:44	0:38:38	1:17:16
4	0:00:06	0:00:29	0:00:58	0:04:47	0:09:34	0:47:47	1:35:34
5	0:00:07	0:00:35	0:01:09	0:05:43	0:11:26	0:57:08	1:54:16
6	0:00:08	0:00:41	0:01:21	0:06:43	0:13:27	1:07:11	2:14:22
7	0:00:10	0:00:47	0:01:33	0:07:44	0:15:29	1:17:22	2:34:44
8	0:00:11	0:00:53	0:01:46	0:08:50	0:17:39	1:28:13	2:56:26
9	0:00:12	0:01:00	0:02:00	0:09:57	0:19:54	1:39:29	3:18:58
10	0:00:14	0:01:07	0:02:14	0:11:07	0:22:13	1:51:05	3:42:10

Approximate Times for an Apple IIGS (fast speed)

# of numbers	1,000 trials	5,000 trials	10,000 trials	50,000 trials	100,000 trials	500,000 trials	1,000,000 trials
1	0:00:01	0:00:06	0:00:11	0:00:55	0:01:50	0:09:10	0:18:20
2	0:00:02	0:00:08	0:00:16	0:01:16	0:02:31	0:12:35	0:25:10
3	0:00:02	0:00:10	0:00:20	0:01:38	0:03:15	0:16:15	0:32:30
4	0:00:03	0:00:12	0:00:24	0:02:01	0:04:02	0:20:06	0:40:12
5	0:00:03	0:00:15	0:00:29	0:02:25	0:04:49	0:24:02	0:48:04
6	0:00:04	0:00:17	0:00:34	0:02:50	0:05:40	0:28:16	0:56:32
7	0:00:04	0:00:20	0:00:39	0:03:16	0:06:31	0:32:33	1:05:06
8	0:00:05	0:00:23	0:00:45	0:03:43	0:07:26	0:37:07	1:14:14
9	0:00:05	0:00:26	0:00:51	0:04:12	0:08:23	0:41:51	1:23:42
10	0:00:06	0:00:28	0:00:56	0:04:41	0:09:21	0:46:44	1:33:28

Note: The use of an accelerator card can further reduce the times. An Apple IIc Plus is slightly faster than the Apple IIGS set at fast speed.

PICK NUMBERS (continued)

Some State and National Lottery Designs

This table contains the design of lotteries that are used in different states and countries. Lottery designs may change occasionally in order to generate more revenue and/or public interest. In lotteries, one must match all the numbers picked in order to win the grand prize (or to split it if there are multiple winners). Prizes can often reach several million dollars in size. Smaller cash prizes are also offered for matching most of the numbers. In some states, for example, if you pick 6 numbers and either 4 or 5 numbers are matched, you also win a cash prize.

Here is an example of how a typical lottery is played. Assume you live in a state where a 6/6/40 lottery is played. This means that you must pick 6 numbers on a ticket containing the numbers 1-40. Your picks are then sent to a computer, and an official ticket is printed with your 6 picks listed. Tickets are sold throughout the state during the week for 50¢ to $1.00 each. At a set time, the sale of tickets ends and the 6-number drawing takes place. This is usually accomplished by a mechanical device containing 40 numbered balls that randomly picks 6 of the numbered balls. If your 6 numbers match the 6 numbers drawn at random by the lottery officials, you are a big winner! It is your responsibility, however, to bring in your winning ticket within a specified time period and claim your prize.

The chances of winning most lotteries are quite slim! For example, your chances of winning a 6/6/42 lottery when buying a single ticket is 1 in 5,245,786! If you simulate a lottery of this type using ***Probability Lab***, be patient and, if possible, use several computers with identical settings and combine results.

State or Country	You Pick	Lottery Picks	Total Numbers
Arizona	6	6	42
California	6	6	49
Colorado	6	6	42
Connecticut	6	6	44
Delaware	6	6	36
District of Columbia	6	6	39
Florida	5	5	39
Florida	6	6	49
Illinois	5	5	35
Illinois	6	6	54
Iowa	6	6	39
Kansas	6	6	33
Kentucky	6	6	42
Maryland	6	6	44
Massachusetts	6	6	36
Massachusetts	6	6	46
Michigan	6	6	44
Missouri	6	6	48

State or Country	You Pick	Lottery Picks	Total Numbers
New Jersey	6	6	46
New York	6	6	40
New York	6	6	54
Ohio	6	6	44
Oregon	6	6	44
Pennsylvania	6	6	48
Rhode Island	5	5	40
Washington	6	6	40
West Virginia	6	6	36
Wisconsin	6	6	54
Lotto America	6	6	54
Oneida (WI)	6	6	36
Tri-State (ME,NH,VT)	6	6	40
Lotto (Canada)	6	6	49
El Gordo (Spain)	6	6	49
Norway	7	7	35
Sweden	7	7	35
West Germany	6	6	49

PICK NUMBERS (continued)

Some Number Problems

1. Do you have a lucky number between 1 and 10? Investigate the chances of your number being chosen and compare it to the other numbers between 1 and 10.

2. There are 100 people at a banquet where a random drawing for two free vacations is held. What is the probability that the youngest person at the banquet will win one of the vacations? That both the youngest and the oldest persons at the banquet will win the vacations?

3. If your state has a lottery, simulate playing the lottery.

4. How much difference does it make to add one number to the total of numbers used in a lottery? Start by simulating a 10/10/10 lottery (a lottery where there are 10 total numbers to pick from and where you pick 10 numbers that match the 10 numbers picked by the computer). Your chances of winning are excellent (100%)! Now change the design of the lottery to a 10/10/11, where you have 11 total numbers to pick from. What happens to the chances of winning? Continue increasing the total of numbers picked from and record your results. Would a 10/10/25 lottery be easy to win?

Appendices

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FLIP COINS (ALL HEADS) **Name** ______________________

What happens to your chance of flipping all heads as you increase the number of coins used?

Before doing the experiment, what do you predict will happen? ______________________

____

____

Directions:

- Flip 1 coin 10,000 times and record how many times exactly 1 head occurs.
- Flip 2 coins 10,000 times and record how many times exactly 2 heads occurs.
- Repeat for 3, 4, 5, 6, 7, 8, 9, and 10 coins and record your findings.

# of coins flipped	Predicted success	Number of successes	Actual success (%)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Summarize your findings: ______________________

____

____

____

Construct a graph with the number of coins being flipped (1-10) on the horizontal axis and the number of successes on the vertical axis.

FLIP 5 COINS

Name ____________________________

When 5 coins are flipped, there are many different possible outcomes. If you were to examine the outcomes in terms of how many heads occurred, you would find there are six possibilities—exactly 0 heads, exactly 1 head, exactly 2 heads, exactly 3 heads, exactly 4 heads, and exactly 5 heads. Which one of these ways (if any) is most likely to occur?

Before doing the experiment, what do you predict will happen? ____________________________

____

____

Directions:

- Flip 5 coins 10,000 times and record how many times exactly 0 heads occurs.
- Flip 5 coins 10,000 times and record how many times exactly 1 head occurs.
- Repeat for exactly 2, 3, 4, and 5 heads and record your findings.

# of heads using 5 coins	**Predicted success**	**Number of successes**	**Actual success (%)**
0			
1			
2			
3			
4			
5			

Summarize your findings: ____

____

____

____

Construct a graph with the number of heads being checked for (0-5) on the horizontal axis and the number of successes on the vertical axis.

SUM OF TWO DICE

Name ______________________________

Rolling 2 standard six-sided dice can produce sums that range from 2 to 12. Are the chances of getting the 11 different possible sums the same?

Before doing the experiment, what do you predict will happen? ______________________________

____

Directions:
- Roll 2 six-sided dice 10,000 times and record how many times a sum of 2 occurs.
- Roll 2 six-sided dice 10,000 times and record how many times a sum of 3 occurs.
- Repeat for sums of 4, 5, 6, 7, 8, 9, 10, 11, and 12. Record your findings.

Sum of dice	Predicted success	Number of successes	Actual success (%)
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Summarize your findings: ____

____

____

____

Construct a graph with the sum of the dice (2-12) on the horizontal axis and the number of successes on the vertical axis.

SUM OF THREE DICE Name ______________________________

Rolling 3 standard six-sided dice can produce sums in the range of 3 to 18. Are the chances of getting the 16 different possible sums the same?

Before doing the experiment, what do you predict will happen? ______________________________

____

Directions:
- Roll 3 six-sided dice 10,000 times and record how many times a sum of 3 occurs.
- Roll 3 six-sided dice 10,000 times and record how many times a sum of 4 occurs.
- Repeat for sums of 5 through 18. Record your findings.

Sum of dice	**Predicted success**	**Number of successes**	**Actual success (%)**
3			
4			
5			
6			
7			
8			
9			
10			

Sum of dice	**Predicted success**	**Number of successes**	**Actual success**
11			
12			
13			
14			
15			
16			
17			
18			

Summarize your findings: ____

____

____

____

Construct a graph with the sum of the dice (3-18) on the horizontal axis and the number of successes on the vertical axis.

SOME QUESTIONS ABOUT *PROBABILITY LAB*

Some experiments ask you to count how many trials it takes before a success occurs. Can this be done using *Probability Lab*?

Yes, but only by using the "step mode" when running your experiment. As soon as the first success occurs, just note the number of trials completed and then exit.

The definition for success that is needed for my experiment is not one of those available. Can I still use *Probability Lab*?

You can still use ***Probability Lab*** to run your experiment, but you will have to do your own success checking and tallying on each individual trial. This will take time to do, but you will gain time in not having to physically produce each trial. If your experiment somehow involves counting specific objects or finding sums with dice, you can set up a success check that gives these outcomes for your convenience and just ignore the success counter.

Another possibility is to set up an experiment that will produce the type of trials needed. Run at least 100 trials and then exit. You are then given the option to view or print a "sample space" of the first 100 trials from your experiment. You can then use these and apply your definition of success.

Some experiments have such a low probability of success that even running 1,000,000 trials might not produce a success. How can these situations be modeled?

There are many situations using ***Probability Lab*** where this can happen. A good example would be a simulation of a state lottery using **Pick Numbers**. If you have a computer lab with several computers, you could set up the same conditions on each computer and then have students combine their results. Be sure in this situation that everybody has identical conditions and have a common recording sheet where each station can record their results.

If you only have a few computers available, you could run as many trials as possible during a day (or let the computer run overnight) and accumulate results over the course of a week or two.

Is it possible to design an experiment in which the probability of success is zero (impossible)?

Yes, every type of experiment has many ways in which this can happen. For example, put 100 red marbles in your bag and define success as drawing out a blue marble.

Is it possible to design an experiment in which the probability of success is one (certain)?

Yes, every type of experiment has many ways in which this can also happen. As an example, you could flip five coins and define success as getting no more than five heads (# of heads ≤ 5). Since this will always be true, the probability of success is certain.

SOME QUESTIONS ABOUT *PROBABILITY LAB* (continued)

Why do some experiments seem to run faster in "auto mode" than others?

There are several reasons why this happens. First of all, experiments using cards or marbles take longer because the cards and marbles are usually mixed up between draws. This is why the counters are updated every 100 trials when reshuffling cards or putting back marbles. The other types of experiments update the counters every 250 trials. Another factor affecting speed is the number of objects being used in the experiment. In most cases, the more objects used, the more time it will take. Finally, the time needed to do some success checks is more than for others.

Is predicting success before running an experiment important?

Prediction is an important part of the problem-solving process because it gets you to think about what a reasonable answer might be before you run the experiment. As you gather information about the accuracy of predictions, estimation can be used in future predictions.

I ran an experiment and there was quite a difference between the actual success and the expected success for the experiment. Is this typical or did something go wrong?

First of all, the number of trials run is an important factor to consider. If you ran the experiment only a few times, this difference should be expected. In some experiments, however, the probability of success is so low that it may take millions of trials before one could feel confident in their findings. In general, one can assume that the more trials run, the closer you can assume you'll be to the expected outcome.

Also, keep in mind that experimental success and theoretical success are two different things. There are other factors that influence the experimental success of an experiment. For example, some people can flip coins a certain way so that heads will occur most of the time. Also, any time that random numbers are used, the true randomness of the outcomes can also be questioned. So one can expect differences to be present.

It is important to try to develop a sense or intuition for both types of success, experimental and theoretical. Realizing that they will vary and discussing the reasons they vary will help students develop this intuition.

Does using replacement (reshuffling cards, putting back marbles) produce different results when compared to similar experiments not using replacement (discarding cards, leaving out marbles)?

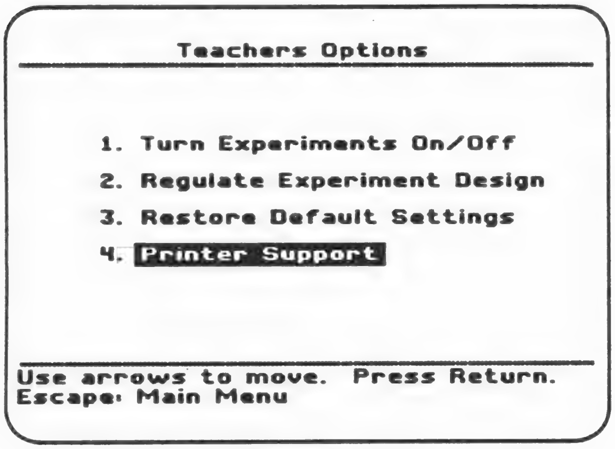
No. The probability of an event happening when not using replacement depends on previous trials and, therefore, can vary from trial to trial. With a well-shuffled deck of cards or a well-mixed bag of marbles, however, the outcomes over the course of an experiment will average out to be the same.

USING A PRINTER WITH *PROBABILITY LAB*

This product is initially set to work with an Apple printer with an Apple printer interface card. It is also set to search for the printer card and use no special printer commands. If you have this setup, you do not need to do anything further.

If your printer uses a different setup from the one described above or if you need to enter special printing commands, you must use the "Printer Support" option.

The "Printer Support" option is found in Teacher Options. To access Teacher Options, type Control-A (hold down the Control Key and press the A key) while viewing the main menu. When you see the "Teacher Options" menu, select the "Printer Support" option (Figure 1). You will then see the Printer Support menu, as shown in Figure 2. The current printer settings are shown at the top of the screen.

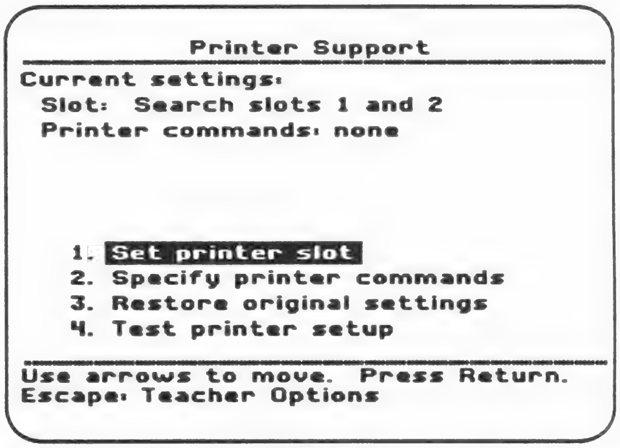


Teachers Options

1. Turn Experiments On/Off
2. Regulate Experiment Design
3. Restore Default Settings
4. Printer Support

Use arrows to move. Press Return.
Escape: Main Menu

Figure 1



Printer Support

Current settings:
Slot: Search slots 1 and 2
Printer commands: none

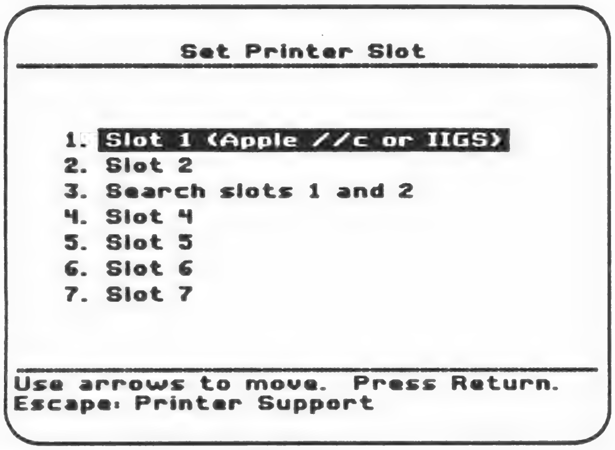
1. Set printer slot
2. Specify printer commands
3. Restore original settings
4. Test printer setup

Use arrows to move. Press Return.
Escape: Teacher Options

Figure 2

Option 1, "Set Printer Slot," enables you to specify the slot number in which the printer interface card is located (Figure 3).

Option 2, "Specify Printer Commands," allows you to enter commands that enable certain types of printers to operate and permits special printing formats. These special commands are listed in the manufacturer's printer or interface card manual. Figure 4 shows the special command you could enter to produce "ultracondensed" printing (17 cpi) on an Apple Dot Matrix, Apple Imagewriter, or Apple Scribe printer. To enter special commands, type the exact characters required. When finished, type ^ (Shift-6) to end.

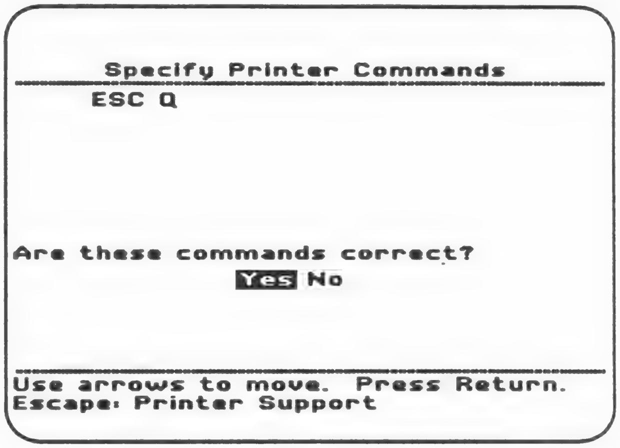


```
              Set Printer Slot

     1. Slot 1 (Apple //c or IIGS)
     2. Slot 2
     3. Search slots 1 and 2
     4. Slot 4
     5. Slot 5
     6. Slot 6
     7. Slot 7

Use arrows to move.  Press Return.
Escape: Printer Support
```

Figure 3



```
         Specify Printer Commands
     ESC Q

Are these commands correct?
                 Yes No

Use arrows to move.  Press Return.
Escape: Printer Support
```

Figure 4

Here are some other frequently used special commands for the Apple Dot Matrix and Imagewriter printers:

Type Style:	Pica (10cpi)	Elite (12 cpi)	Bold
Command:	Escape N	Escape E	Escape !

Do not set up your printer to use a proportional font. This setting will cause printed reports to be formatted incorrectly.

Option 3, "Restore Original Settings," returns all printer settings to their original state. The original settings are to search slots 1 and 2 and not to use any special printer commands.

Option 4, "Test Printer Setup," prints out all of the keyboard characters. If these characters are not printed correctly, check the settings on your printer, check to see whether your printer has been connected correctly, or look at your interface card manual for special commands.

All changes made to the Printer Support settings are saved on the disk and are permanent until you use the Printer Support option again to change the printer settings.

CREDITS

Probability Lab was produced by a MECC development team that included Brian Nesse, Jane Peterson, and Craig Solomonson.

Special thanks are extended to the following teachers and their students for assisting with the classroom testing of this package:

David Jaeger and Bill Kast
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TO THE READER

MECC has made every effort to ensure the instructional and technical quality of this courseware package. Your comments—as user or reviewer—are valued and will be considered for inclusion in any future version of the product. Please address comments to:

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